FEDERAL UNIVERSITY OF ITAJUBÁ
POST-GRADUATION PROGRAM IN INDUSTRIAL ENGINEERING

METHODOLOGY FOR PROJECT PORTFOLIOS SELECTION USING MULTICRITERIA OF THE CAPM, SEMI VARIATION, AND THE GINI RISK COEFFICIENT

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Concentration area: Industrial Engineering.

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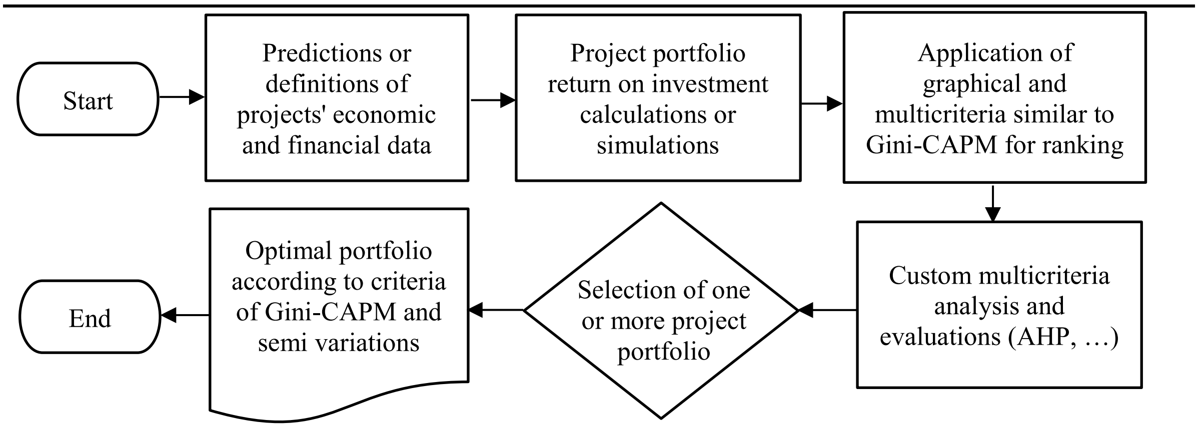
Itajubá
2022

ABSTRACT

Criteria from Gini-CAPM and of Gini-semi-variations metrics are good options to compose methods for projects portfolio selection. The adequacy is even more, especially when considering the trade-off between return and risk and the covariations in the adjacent selection. These methods can help significantly because they have more robust risk coefficients for assessments of non-normal probability distributions, which are very common in projects portfolio selection. However, searches for methods that meet the selection needs using the adjacent criteria are unsuccessful, including for projects of renewable solar energy generations using cellular photovoltaic panels, which have stood out among the options. Thus, this work seeks to help minimize the gap by presenting methods for selection using criteria from Gini-CAPM and of Gini-semi-variations, and with significant novelties. Historical and simulations data stochastic evaluations indicate that the portfolios selected by the methods are attractive options for implementations. These portfolios have reasonable probabilistic expectations of the trade-off between risk and return and satisfactory protection to avoid mistakes caused for not considering covariations in return on investment. These are significant advances on the current knowledge frontier and will likely allow the increased use of the concept. The methods also present theoretical contributions in adaptations of the benchmark models, which help to minimize the adjacent literary gap, in addition to a financial class structure as it considers most of the scenario variables.

Graphical Abstract



KEYWORDS: Project portfolios selection; Gini-CAPM and Gini-semi-variation; Projects selection considering covariations; Photovoltaic solar energy microgeneration; Social welfare.

Nomenclature

α_j	Project *j* participation in the portfolio
B	The upper limit of initials investment
C_{sj}	Matrix with all binary combinations of portfolios s and projects j
$F(r_j)$	The cumulative distribution function of project or portfolio j return
GB_s	Vector of non-diversifiable Gini risk of all portfolios s
GP_s	Vector of estimated Gini-price of all portfolios s
GS_s	Vector of maximum excess return per Gini-risk unit of all portfolios s
ij	Projects or portfolio *j* in period i
P_{ij}	Matrix of investment required for each project j in the period i
L	The lower limit of acceptable returns
p	The portfolio total number
s	Identification vector of all portfolios
r_{ij}	Matrix of return of the project or portfolio j in the period i
R_e	Return or price of the project or portfolio e
R_f	Risk-free rate
R_m	Return of the market portfolio
$\Gamma_{jj'}$	Matrix of Gini correlation between projects *j* and *j'*
x_j	Decision variables vector to each project *j*
z_j	Vector of maximum relative participation of each project *j*
Δ_j	Gini risk of project or portfolio j

Abbreviations

AHP	Analytic Hierarchy Process
CAPM	Gini Capital Asset Pricing Model
ISI	Institute Scientific Information
MG	Mean-Gini
MPT	Modern Portfolio Theory
MRA	Minimum rates of attractiveness
MV	Mean-variance
PERT	Program Evaluation and Review Technique
PPAS	Project portfolio analysis and selection
ROI	Return on investment
SELIC	Special Settlement and Custody System
SEPC	Solar energy by photovoltaic cells

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SUMMARY

1 INTRODUCTION

In this Section, we seek to elucidate the main guidelines of this research, and for that, we present the main subject or theme of the study and its subthemes; the identified problems and their relevancies; the justifications and motivations to help solve the issues; the general objective and the specific objectives and; the structure planned for the study.

1.1 Central Theme and Subthemes Contextualization

Project portfolio analysis and selection (PPAS) is the central theme of this research. In these times of fierce competition due to the worldwide market opening, business management's excellence has become absolutely necessary. For part of the business models, this excellence will only guarantee permanence. Furthermore, the constant increase in the excellence of project management will allow the achievement of several benefits to the business, such as: maximizing capital returns, minimizing the risks of the activity, maintaining or increasing the competitive position, allocating resources in general in a rationalized way, and others (ISAIAS et al, 2021; SONG et al., 2019).

According to Marija et al. (2015), the stage of selecting portfolios in project management is critical to achieving the desired business benefits. The reason is that activity is fundamental and one of the most important in project management. Its execution with excellence generally results in constant beneficial increments for the business in all senses. Moreover, although this will not always happen regarding all the resources used in project conceptions, there is usually at least one that restricts the system by its scarcity. Therefore, it is possible to affirm the absolute importance of analysis and selection in project portfolio management (LI et al., 2020; NIÑO et al., 2015; SANTOS et al., 2018).

The methods used to analyse and select assets in general (including project portfolio) classify as economic or financial. Regardless of any of the classifications of the methods, they are strategic for the activity. Thus, companies, institutions, and other entities that develop projects can lose their competitiveness considerably if they do not have any of these types of methods to help them. Still, research characterisation can derive studies in many thematic. Therefore, other sub-themes are objects of study with emphasis on this research, always seeking knowledge increments for applying the sub-themes to increase the scientific domain of the central theme. Between these secondary topics, we have portfolios of solar photovoltaic generation projects, the Capital Asset Pricing Model (CAPM), the Gini risk coefficient (also known as the Gini coefficient, which, in turn, may have some advantages in applications), among others (ISAIAS et al., 2021; RYDER et al., 2020).

The world is being held hostage to non-renewable energy sources, which is a significant problem. As the resource is limited, it has become the centre of conflicts in several segments: economic, market, ideological, military, among others. Moreover, with the advance of demanding technologies and the population in general, energy consumption is significantly increasing, which will cause the reserves of non-renewable sources to end quickly (MAIER, STREET and MCKINNON, 2016). The growing energy demand is undoubtedly one of the significant challenges today. Its segmental conflicts generate dire consequences for society. Therefore, the constant change in the world energy matrix towards more excellent renewable sources is desirable, especially if the fossil and nuclear options have significant reductions (GAWEL et al., 2017; ISAIAS et al., 2021; WEIDA, KUMAR, and MADLENER, 2016).

Among renewable energies sources, solar photovoltaic stands out for its great potential due to the increasingly affordable cost of implementation. It also is clean, sustainable, and has a significant margin for generation and growth (AGOUA, GIRARD, and KARINIOTAKIS, 2018). In renewable sources, solar energy by photovoltaic cells (SEPC) has been the first option of the current society to correct the energy matrix. The choice is mainly due to the significant benefits of the technology. Also, it is essential to note that they are among the most sustainable, do not emit pollutants or use scarce raw materials, reduce noise pollution, can last up to two and a half decades at least, and have a considerably reduced payback (FARIA and TRIGOSO, 2017; LEE, HONG, and KOO, 2016).

However, as well as for project selection in general, the SEPC project portfolio analysis and selection (PPAS) is still deficient. In other words, we found some models to perform the selection, but they are few, do not consider the risk and return trade-off, do not consider the covariations (nor the most important, of the risk of return on investment - ROI), among others (STOJCETOVIC et al., 2016). The covariations (also called interdependence) between projects can occur in any input or output parameters in the adjacent portfolio, simply the formation with two or more components (LOUREIRO, GOLDMAN and NETO, 2018; SANTOS et al., 2018).

Among several options tested as alternatives to variance, the so-called Gini risk coefficient stands out. Its concept is as intuitive as that of standard deviation. Moreover, it is mathematically a more robust metric to applications in non-normal distributions, principal due to the structure of its probability density function (DANG et al., 2021; PMI, 2017). The Gini coefficient can replace the standard deviation to minimize the main obstacle for applying CAPM in non-normal distributions, and this development is the so-called Gini-CAPM (OGWANG, 2016). The Gini risk coefficient has stood out in substitutions for the variance's risk values, where its main derivative is the standard deviation. The Gini risk is equivalent to the standard deviation. One of the first models developed using the Gini risk is the so-called mean-Gini model (MG) (ISAIAS et al., 2021; MARCONDES et al., 2017).

The Gini Capital Asset Pricing Model (Gini-CAPM) is an appropriate methodology to contribute to the presented questions solutions. The model has several benefits when used for asset portfolio management. Because it derives from the Modern Portfolio Theory (MPT), the methodology fully considers the interactions between portfolio assets. These interactions, in turn, can strongly influence the results. The Gini-CAPM mainly arose because the traditional CAPM risk measure, the variance, which is a highly efficient metric, is the most used in numerical methods, but it is significantly less appropriate for non-normal distributions. The reason is that in PPAS, the assets have non-normal random variables and parameters (MARCONDES et al., 2017; SUKONO, 2018).

1.2 Problems Contextualization

The problems highlighted for this research are those which hinder the sub-themes used to increase the central theme's technological domain. Among these problems is the absence of models or methods similar to Gini-CAPM, but for applications in evaluation to project portfolio selection, mainly in terms of solar energy by photovoltaic cells projects, which have strong appeal in several areas. Another problem is the restriction of CAPM models or methods application only where the adjacent data come from normal distributions. Furthermore, finally, it is the absence of developments of the second level that can become possible methods with criteria of the Gini-CAPM and semi variations for project portfolio selection.

In short, studies on the CAPM methodology and of semi variation allow stating that they could be beneficial in the project portfolio selection. In this case, their implementations in the segment have great potential (NHLEKO and MUSINGWINI, 2016). The conceptual basis of risk in the mean-variance (MV) model (that has a central logic, the search for the optimal trade-off between mean return and risk) was the main barrier to developing a method similar to that of CAPM to projects, but this only until the development of Gini-CAPM. The reason is that although its concept is quite intuitive, the MV carries the restrictive normality of adjacent distributions. Also, "normally distributed data is generally not seen in the project portfolio. In this case, the distributions are usually binomial, triangular or beta, among others" (ISAIAS et al., 2021).

Another complicating factor in PPAS is the interference caused by independence between projects. According to Zhang (2016), interdependencies may distort the results of the portfolio parameters of the projects under study. Furthermore, if it does, the distortion should influence, for example, revenue, return, risk, techniques, resources, costs, investments, and others. The models generally used to estimate interdependencies between projects consider values in scores for the various parameters studied, and these estimates are performed simultaneously for all the parameters. In sequence, a matrix with a two-dimensional will house the values.

Multidisciplinary teams of specialists usually define the interdependence mapping scores. They have their initial results in linguistic variables that, in turn, should undergo further transformations for numerical data (by Set Theory, Logic Fuzzy, and others.). And yet, the final prioritization decision is often aided by sophisticated technologies such as AHP and Delphi, among others (GARCIA and CASTRO, 2018; ZHANG, 2016).

However, research-based in the Institute Scientific Information (ISI) and the Scopus database reveals more structural gaps in this research's interest theories. After literary consultations, we found the absence of methods, models, procedures, or structured programs to generate triangular distributions correlated by the Gini parameter. Therefore, the relevant need for developments in this direction is evident.

Other research in the Institute Scientific Information (ISI) and the Scopus database reveals allows identifying more gaps in theoretical developments regarding the parameters of applying the Gini coefficient or of semi-variation for triangular distributions. The fact occurs that even this type of distribution has a trivial mathematical description and "a very high degree of use in project portfolio management" (PMI, 2017). Thus, after the arguments provided in the paragraphs preceding, it is possible to elaborate on some research questions:

> *"How to develop, structure and, validate methods, models, programming, or procedures, that use Gini-CAPM and semi variation criteria for applications in PPAS (including in the scenario SEPC) and that is still able to consider interdependencies?"*

Furthermore, from this central question, other underlying ones emerge: How to adapt the methods, models, programming, or procedures developed to answer the central question to deal with interference caused by interdependencies (at least that of ROI risk) and for applications in PPAS (including SEPC, not just from Brazil, but from all over the world)? How to adapt the methods, models, programs, or procedures developed to answer the firths questions and still reduce the computational cost of the application in PPAS due to dispensing with extensive simulations? What are the needs to sustain or enable planned developments to answer the previous questions?...

1.3 Justifications for Research

The justification for the exercise of science must reflect the reason for the research, and this must be of great importance for a society, an academic community, a professional category, and others. Technological stagnation in the PPAS activities of companies, institutions, and others will likely impose financial difficulties due to the high competitiveness and marked gradual loss and the constant deterioration of the competitive position by inefficiency (LOUREIRO, GOLDMAN and NETO, 2018; ISAIAS et al., 2021; SONG et al., 2019).

According to Institute Scientific Information (ISI) and the Scopus database, we justified this research from a scientific perspective by the high relevance and current trends of its themes. Furthermore, another is the probable novelty of combining these themes that, in turn, should contribute to publications in relevant scientific journals. Therefore, between the main justifications for this research is help the large slice of the business sector to which project management is essential seeking out maximizing capital returns; minimization of the risks of the activity; maintenance or increase of the competitive position; allocation of resources in general in a rationalized way; and others.

Also, the need to increase the excellence in business management projects is latent and relevant. Moreover, this research mainly concerns the analysis to select portfolios in a structure to try to maximize benefits. Another significant justification is that the literary gaps materialize due to the absence of methods, models, procedures, or structured programs to use this research's sub-themes to increase the central theme domain. Moreover, technically, in a post-graduate program are important the thesis generated and the publication in scientific journals, besides another's potentials as pseudocodes or algorithms, pseudocodes in Python, software, patents, among others.

In fact, we can be described the thesis of this as "it is possible to plan, design and implement more robust methods against non-normal distributions in PPAS, including SEPC (not just from Brazil, but from all over the world), capable of extending the benefits of CAPM and semi-variation metrics for the scenario". These in addition to other possible contributions such as interdependencies considerations in the ROI, reduction of the computational cost of adjacent applications, besides to needs to support and/or enable planned developments, and with significant novelties.

1.4 Research Objectives

In macro form, this research seeks to develop structured multicriteria methods using as main benchmarks the Gini-CAPM and metrics of Gini-semi-variations. Furthermore, in these developments, we always sought significant concepts to we have more chances which the academic and scientific community to accept the methods to project portfolio selection.

In this sense, we can determine the objectives of the research, where they should seek answers to the problematization questions presented:

a) Develop a first structured method for project portfolio that analyzes and selects using multicriteria similar to Gini-CAPM tools;

b) Develop a second method with the same structure and capacity as the first, but that considers interferences caused by interdependencies between projects (at least in terms of ROI risk);

c) Develop a third method, where again it should present the structure and capacity of the first two and reduce the computational cost of the application by dispensing with extensive simulations;

d) To adapt the structure of the methods developed so that we can apply them to the selection of photovoltaic solar energy generation project portfolios;

e) Develop secondary-level methods to support or enable the three first.

Thus, with this study, we expect to contribute scientifically and significantly, seeking to solve identified problems and minimize the literary gaps found. In summary, the proposal is a set of methods, where we structured them for applications in PPAS, using multicriteria of the Gini-CAPM and of covariations, considering the possible Gini-covariations of ROI, and in an innovative way. We organized the study to present as follows: Section 2 shows the essential concepts used for the design of the methods; Section 3 presents the materials and methods used in conducting the study; Section 4 presents the method application and results discussions; and finally, Section 5 presents the major research conclusions

2 LITERATURE REVIEW

In section 3, where we will present the developments of the methods, there will also be other brief reviews necessary to understand the developments themselves. Thus, this review section is summarized and presents: Standard Deviation Versus Gini Risk in Portfolio Evaluations; CAPM Using Standard Deviation and Using Gini Risk; Contemporary Project Portfolio Selection; Gini Correlation Coefficient; Projects ROI Variations and Covariation; Portfolios Selection by Maximum Excess Returns Per Risk Unit; Project Portfolios Selection by Non-diversifiable Beta-Gini Risk; Project Portfolios Analysis and Selection by Gini-CAPM Pricing; and, Literary Gaps.

2.1 Standard Deviation versus Gini Risk in Portfolios

The customized optimization of the trade-off between the mean and standard deviation of Markowitz (1952) was revolutionary for its time and is the main foundation of the Modern Portfolio Theory (MPT). In this theory, a portfolio is inefficient if there is another one with a greater return and less or equal variability, or less variability and greater or equal return. Furthermore, in these analyses, the Markowitz portfolios consider a variance to measure the risk (GARCIA, and CASTRO, 2018; KUMAR et al., 2018).

The coefficient Gini derives from the so-called "mean-Gini-difference," proposed by Corrado Gini in 1912 as an alternative measure of variability. This risk metric application was as statistical dispersion to verify income inequality. Lately, the Gini risk has several applications in other areas, but always to measure dispersion (HUANGBAO, 2014; NUTI et al., 2015; PARSA, DI CRESCENZO, and JABBARI, 2018; ROGERSON, 2013). The mean-Gini (MG) model is less restrictive than the mean-variance (MV) model because it does not depend on a quadratic utility function on the adjacent trade-off, nor does it have inductions for some probability distribution (MARCONDES et al., 2017; REBIASZ, 2013).

Another strategic advantage in using the MG model is the definitions of the necessary condition of stochastic dominance of second order. For example, if a first one has a higher return in evaluating two portfolios, it will have stochastic dominance over the second if, and only if, its risk is less than or equal (ROLAND, FIGUEIRA, and DE SMET, 2016). In fact, in comparisons to define stochastic dominance, we have equations of inequalities. However, if only one inequality occurs, the other can be established as equality for these conditions to be sufficient. In addition, the portfolios' respective cumulative distributions can have at most one intersection (RINGUEST, GRAVES, and CASE, 2004).

In summary, the use of Gini presents some advantages in project portfolio evaluations: it is simple and easy to understand, it allows the construction of groups of portfolios that meet the necessary conditions of second-order of stochastic dominance (return and risk analysis), it applies to all risk-averse decision-makers without requiring explicit knowledge of the utility function (with variance must be necessarily quadratic; and the greater the risk, the greater the aversion), and it does not presuppose nor is it linked to a specific type of probability distribution (MARCONDES et al., 2017; NHLEKO, and MUSINGWINI, 2016).

Thus, the use of the Gini risk in place of the standard deviation can be justified because, even though the standard deviation is a consecrated risk metric, it has some limitations. For example, the use of variance can induce equal weights to positive and negative deviation values, leading to wrong conclusions. Among the types of distribution possibilities, the authors recommend the use of triangular. The reason is that this type can use the so-called three-point techniques to define its parameters, and, therefore, the estimate occurs in a way more efficient and direct (PMI, 2017).

To exemplify the aforementioned strategic gap for this research, suppose a decision-maker facing a scenario where one needs to choose between two portfolios of projects P_i and P_j and which they were previously selected, among other options, by a hypothetical company multifunctional.

On the assumption also, the decision-maker has information that portfolios have an estimated standard deviation of 1.00% of the monthly return, and they also count the monthly variation of return between - 2.00% to 4.00% to a confidence level of 99.73%, and by coincidence P_i and P_j have a respective predominance of projects for the national and international markets.

Because of the coincidence of standard deviations and confidence intervals of the two investment options, then the decision-maker assumes could simulate only one probability distribution for the two portfolios. Furthermore, induced by the informed standard deviation parameter, the decision-maker also supposed normality for the distribution. In Figure 1, the first graph represents the probability distribution for the two variables. In this case, the decision-maker will have a great tendency to implement P_i in order to strengthen the national market. This based on the estimate that both projects have approximately 69% probability of having monthly returns greater than 0.5%.

However, the choice of P_i would be an error because the induction decision-maker to use a normal distribution for the P_i and P_j due to the standard deviation parameters is present in the analysis. But, when verified with attention, it would identify that the probability distribution of P_i is skewed, according to the second graph in Figure 1, and the variable would have only approximately 30% of probabilities of monthly returns greater than 0.5%. The error would be more significant if the variable P_j is also biased but in the right way. For example, this could occur because the decision-maker estimated the probability distribution of P_j as well as that of P_i from adjacent triangular distributions. In this case, the displacement of P_j, for example, due to high dollar estimates favouring projects for export to be positive. The bias would allow us to evaluate in this hypothetical case that P_j would have monthly returns greater than 0.5% with a probability of approximately 92%, as shown in the third graph of Figure 1.

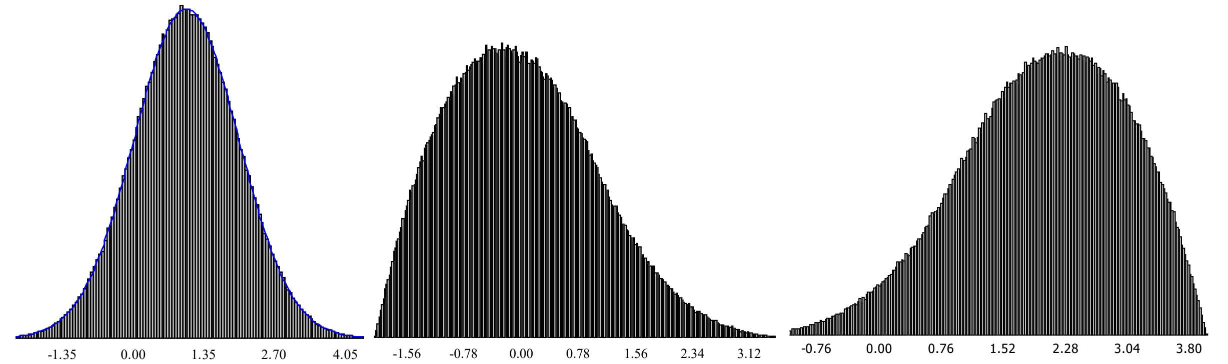

Figure 1 – Estimated distribution of portfolios

Source: authors

The didactic example clarifies the possibility of inducing misunderstandings that variance can generate by attributing equal weights to positive and negative deviation values. In turn, these assumptions can lead to erroneous conclusions, and even worse, lead to strategic errors with significant consequences. If the hypothetical decision-maker used the Gini risk as a metric in this hypothetical didactic example, the error would not occur. Another feature that helps avoid the adjacent error in using the MG model is its very efficient cumulative probability density function. This function derives from a rank known as the cumulative probability density for samples. Therefore, we concluded that using the Gini risk, the misunderstanding would have less probability of occurring.

2.2 CAPM using Standard Deviation and Gini Risk

Authors have occasionally raised questions if CAPM and its risk metrics are not adequate to measure portfolios' performance. In this sense, the most cited justification for this questioning is always the need for the excess of adjacent returns to have a normal distribution, which usually does not occur. Based on the traditional CAPM questions, several developments have emerged trying to complement it. For example, the L-Performance model which is a proposal of a new performance parametric index (HOMM, and PIGORSCH, 2012; ZAKAMOULINE, and KOEKEBAKKER, 2009).

The index has been presented as an alternative to CAPM tools and, the central argument was an inability to consider asymmetries of the probability distributions (OKUNEV, 1992). The L-performance technically resembles the Sharpe index and is the ratio between the first-order moment and the second-order moment. However, in this case, adjusted moments are used instead of conventional ones. The adjustment parameters allow the L-Performance index to indicate specific risk levels and financial criteria (DAROLLES, COURIEROUX, and JASIAK, 2009).

According to Okunev (1989), empirical tests evaluating the Gini-CAPM methodology cannot find significant differences compared to the traditional CAPM. Trying to explain the result, the authors present some possibilities: at least for the sample data of the test, the Gini-CAPM model can differentiate only at specific points; It does not exist the expected competition among the studied portfolio, at least not at the supposed level; and others (OKUNEV, 1992). The authors Auer and Schuhmacher (2013) perform tests on investment fund portfolios and, the results allow us to affirm that Sharpe's Asset Pricing Methodology is indeed reliable. Furthermore, this occurred in a very volatile scenario and considering assets with non-normally distributed returns.

Darolles and Courieroux (2010) presented a model that proposed the insertion of a conditional adjustment variable in the Sharpe index to classify investment funds. In this research, a battery of tests was performed, varying the presupposed information considered in the model's calculation. This study concluded that the classification with the Sharpe model presents essential advantages for the investor: the performance measures correspond to the standard measurements that used; the numerical results will be available for comparisons, even if obtained with different assumptions; the classifications based on regression analyses are easy to calculate; and others.

The author Oussama (2020) presented a method for estimating stock prices in emerging markets using the CAPM methodology regression. The author proposed an optimal portfolio for four emerging stock markets based on the mean-variance trade-off model and the capital asset pricing model. Later, they compared the results with the German market to understand possible differences. Oussama (2020) shows that, for most stocks, CAPM explains asset prices well. It is also important to note that, however, for some stock market shares, the behaviour is different between the short-term and long-term scales, which is consistent with the hypothesis tested by the research.

Another interesting study for this research on Gini-CAPM is in Shalit and Yitzhaki (1984). In this research, the authors state that Gini-CAPM is an excellent option to overcome traditional CAPM's restrictive assumptions. In particular, in this research, the risk is measured by the Gini coefficient as an alternative to variance. In this case, similar to what occurs in traditional CAPM, the investors should find the market portfolio for later strategic uses of it. Indeed, they propose using Gini risk in the portfolio's analysis and selection.

Another critical piece of information is that since the efficient Gini-CAPM portfolio meets the necessary conditions of second-order stochastic dominance, it does not require that the probability distributions of returns under study are normal. Furthermore, another attractive feature of the model is that it applies to probability distributions of returns whose first and second moments exists (MARCONDES et al., 2017; OKUNEV, 1992).

Okunev (1988) presented a comparative study between the mean-Gini model and the mean-variance model in portfolio selection. This study fundamentally assumes that the methods traditionally used to compare uncertain perspectives are mean-variance models and stochastic dominance. Still, in this case, the author based himself on Shalit and Yitzhaki (1984), where they proposed an alternative model based on the mean-Gini to compare uncertain perspectives.

According to Shalit and Yitzhaki (1984) and Okunev (1988), the mean-Gini model is similar to the mean-variance model, and it uses a two-parameter statistic to describe the probability distribution of risky returns. Theoretically, the adjacent model is consistent with investors' behaviour in uncertainty conditions for a broader probability distributions class. As a result, he generated the efficient frontier of mean-Gini and, later compared with the mean-variance's efficient frontier. As one of the main conclusions, the author stated that for the sample data, the mean-variance model presented results close to those observed in the application of the mean-Gini model, but the latter had a higher computational cost.

Strategically according to the proposal of this research, Gemici-Ozkan et al. (2010), Ringuest and Graves (2005) and, Ringuest, Graves, and Case (2004) use the coefficient of Gini as a dispersion metric to a selection of project portfolio (MARCONDES et al., 2017). In Shaffer and Demaskey (2012), the authors consider less restrictive characteristics in evaluating a portfolio, where the Gini coefficient is a risk measure in studying currency hedging operations. They proposed selecting a project portfolio using the mean-Gini model for uncertain scenarios. According to the authors, there is a wide variety of models of project portfolio selections in the literature.

On the other hand, Marcondes et al. (2017) claim that organizations still have great difficulties in this selection to diversify the portfolio in search of better results. Also, using models of mean-Gini and stochastic dominance to select projects has gained significant attention because until then, these models do not consider uncertainties regarding the project parameters.

The main criticism of the traditional CAPM, on the other hand, is related to its applications in non-normal probability distributions. These applications often receive critics because, according to research, decision-making mistakes caused by assumptions regarding the adjacent probability distributions. Nevertheless, in fact, the criticism cited is the central gap to justify this research. This is because the estimated distributions do not need to be normal for this research.

Gini-CAPM's main criticism is that the methodology's theoretical benefits are not in all cases evident in empirical tests. However, the nature of these empirical tests is passive to questioning in several of their dimensions. For example, was the data from the studied probability distributions sufficiently skewed that Gini-CAPM could show its potential? If the answer to the question present is no, the Gini-CAPM model, at minimum, offered more excellent protection to investors. Therefore, the Gini-CAPM would offer more security into the investor's decision if there are biased distributions.

2.3 Contemporary Project Portfolio Selection

Shafahia and Haghanib (2018) propose a model to analyze, select and program when projects allow implementations in phases. This model is of mixed and integer programming (MIP), seek to maximize the net present value (NPV) of future investments and, considers possibilities for reinvestment. In this research, the model showed itself ideal for determining each phase's implementation solution, and it considers the dependencies between different stages and answers the needs of future phases' developments.

In the research, the model solves problems with many projects or phases, but the size is initially smaller. Then, it is a model with a two-step heuristic, where the first add projects to the selected set based on the gains obtained. In the second, some phases of the chosen projects are eliminated based on the probability of success. Then, sensitivity analyses are performed, changing various parameters that affect the performance of the heuristic, such as, for example, different measures of gains and different initial budgets.

Also, in Shafahia and Haghanib (2018), the results are favourable for the pre-processing stage and of solution heuristic. In small scenarios, heuristics can find the ideal MIP solution in almost all cases. However, for large designs, the heuristic finds solutions within approximately 100 seconds, which is much better than competing models in the literature.

In Tofighian and Naderi (2015), the research considers an integrated bi-objective problem of project selection and programming. The model optimizes the total expected benefit (depending on time) and the variation in resource use. Although this modelling structure has become entirely researched, the algorithms available for this purpose have some deficiencies compared to the model proposed. The model is linear, integer, and mixed and uses a configuration of ant colony algorithm and the Pareto boundary as a verification mechanism to evaluate the proposed algorithm, to comparisons with the genetic and dispersed search. Finally, using comprehensive numerical experiments and statistical tools proves that the proposed model's optimization surpasses the alternatives.

Hasuike, Katagiri, and Ishii (2009) state that many studies propose portfolio models based on stochastic and diffuse approaches. In this context, the models consider random and ambiguous conditions and, few of them performed well in their respective applications. Therefore, to overcome this deficiency, the diffuse, random, multi-objective and, CAPM-based model was proposed. Still, in the research, the authors state that their mathematical programming model presents applicable definitions for mapping the scenario's randomness and uncertainties. They complement this research by defining some criteria for introducing restrictions to transform the problem into deterministic programming. In this way, it is possible to build a method to obtain an optimal global solution that restricts the search for current technologies for selecting project portfolios to those with some bias of the trade-off between mean and risk in Projects. By the scope of this research, it is possible to observe a wide gap of opportunities here.

According to Peres and Gomes (2016), decisions generally have restrictions on the budget in selecting a project portfolio. Therefore, it is necessary to choose the portfolio to meet restrictions and ensure growth. The authors address the problem by proposing a multi-objective, binary and, nonlinear mathematical model. This model considers all the most important factors mentioned in the literature related to portfolio execution and selection.

Still, in the aforementioned research, the authors state significant uncertainties in different decision-making aspects and that the ideal strategy is to incorporate some parameters into the Fuzzy model. The intention is that these parameters allow representing information not fully known by the decision-maker. Therefore, the resulting model receives the classification as fuzzy and multi-objective and also generates graphical tools, which show its usefulness to assist in the decisions in question.

Huang, Zhao, and Kudratova (2016) present a joint problem of project selection and programming, where experts have estimate initial disbursements and net cash inflows from projects lacking historical data. The new model is of mean-variance and mean-semi-variance and also considers the relationship and the order of temporal sequence. Still, in the research, they solve complex problems, where they propose methods to calculate partial semi-variance with low uncertainty and to calculate partial semi-variance with high uncertainty. This is through an artificial intelligence algorithm that is also hybrid, where the model integrates genetic and cellular automation. The research presented two examples to illustrate the application and the meaning of the new model proposed.

According to Marcondes et al. (2017), there is a wide variety in the literature of models to select project portfolios. On the other hand, organizations still have great difficulties in diversifying the portfolio searching for better results. Besides, the mean-Gini model and stochastic dominance to select projects have gained significant attention in the literature. These models do not consider uncertainties regarding the projects' parameters in this selection. They also present a model for project portfolio selection through an approach encompassing the trade-offs mean-Gini, stochastic dominance, and impact of parameter uncertainties. Also, this research uses Monte Carlo simulation to assess the effects of parametric uncertainty on the selection. The results show that the influence of uncertainty is significant in the efficiency of the selection.

Sukono et al. (2018) state that correct investments are economic growth factors in countries in a scenario with correct net investment to projects. However, when determining this type of investment, maximum returns with a minimum risk level are utopia. Thus, the authors state that it is necessary to know how to allocate capital to provide the ideal benefit. They also discuss equity investment based on CAPM, where the prediction beta parameter occurred using two approaches. The first is the concepts of covariance, and the second occurs through the optimization of genetic algorithms. The model also assumes that the data analysed meets the requirements of the CAPM. The research results by Sukono et al. (2018) show that estimating the beta parameter using the covariance approach and genetic algorithm leads to identical decisions. Furthermore, they concluded that the results used as a counterpart for investors who buy at low prices and sell at high prices are great.

It is essential to clarify that this interdependence is a risk form. Interdependence between projects is the interference, positive or negative, that can occur in any of the portfolio parameters, only by selecting its components in at least pairs, that is, $i \geq 2 \ldots n$ (BARTOSOVÁ, MAJERCAK and HRASKOVÁ, 2015; BHATTACHARYYA, KUMARB, and KAR, 2011). More precisely, the interdependence variable occurs when the result of any of the portfolio parameters is different from expected, according to the individual result of the parameters in question (ZHANG, 2016).

The interdependencies between projects can occur in the parameters of revenue, return, risk, techniques, resources, costs, investments, financing, financing rate, and others. Nevertheless, as happens among the other portfolios' parameters, interdependencies can be strongly related to each other. It is also important to mention that certain interdependencies can be components of others (ZHANG, 2016). Due to historical data's absence, it generally uses the other project parameters' prediction techniques to obtain interdependencies values, for example, analogues, from specialists, bottom-up, pert, parametric, and others (RUNGI, 2010).

In general, models countable these interdependencies in scores by parameters in a two-dimensional matrix (although interdependence can also occur when $n > 2$). These scores are usually defined by specialist teams that indicate the interdependencies in linguistic variables. Subsequently, the variables receive transformations by theories as logic fuzzy, and others (TASEVSKA, and TOROPOVA, 2010). Ranking tools can improve the interdependence matrices values after the linguistic variables' transformations, as AHP and Delphi. These tools will increase the resulting models' solutions by the personalized aggregation of importance to each interdependence (LOUREIRO, GOLDMAN and NETO, 2018).

Li et al. (2016) developed an extended model for the project portfolio selection problem using several periods, where the model incorporates the interdependence factors of the projects while aiming at real-life applications. In this case, in choosing the best execution schedule for the projects, divisibility is a strategy, not an event. Thus, the model extends the divisions to the classic concept of "interdependencies between projects" and still consider additional reinvestment restrictions, installation cost, cardinality, precedence relationship, and programming. Finally, for project efficiency calculations, the model derives a representation of mixed-integer linear programming. The research also presents a numerical example in four scenarios to illustrate, and it presents for the first time the positive effects of projects' divisibility.

According to Soofifard and Gharibb (2017), large projects' risks are natural and inherent characteristics and generally are considered independently in the analysis, but in most cases, the risks are dependent on each other. Thus, the research proposes a model for selecting responses to portfolio risks considering dependencies, intending to optimize the defined criteria. Therefore, the model considers the relationships between responses to different risks. In conclusion, they point out that the lack of consideration or evaluation of the interactions among risks increases projects execution costs. Thus, the adjacent research model can optimize different criteria in an objective function, using the multi-objective harmonic search to obtain solutions.

Zhang (2016) states that when analysing risk responses, often we assume that they are independent. However, the risks in a project affect each other and, independence between them rarely exists. Thus, this research provides an approach to measure risk interdependence of a quantitative form and have based on an optimization model to select response strategies considering the expected loss, interdependence, and directions. There are two main findings from this study's analysis. First, the expected utility would be more sensitive to the interdependence of risks than to its directions. Second, the lack of attention or neglect of interdependence between risks will reduce the expected utility and increase implementations cost.

2.4 Gini Coefficient

Researches reveal that MG concepts have low application rates compared to MV concepts. These low rates occur, among others, due to the more significant effort for the MG calculations – which cannot be a plausible justification nowadays, due to all contemporary computational evolution (MARCONDES et al., 2017). Technically, among several definitions of the Gini difference, the most used is half the expected distance between two realizations of the same random variable. From the definition, it is then possible to sketch a formula that allows t to calculate the Gini coefficient later. The Gini difference formula is in Equation (2.1) (MARCONDES et al., 2017).

In Equation (2.1), the parameter r_i and the parameter r_j represent two realizations of the same random variable r. The equation in question expresses in a very intuitive way the half of the expected distance between two random variables of the same distribution obtained in pairs (BUKOVSEK et al., 2021). Although Equation (2.1) is quite intuitive, its solution is not trivial. It will require, in addition to the definition of the function $f(r)$ that explains the random variable r, the application of a definite integral in this same function starting at r_i – if r_i < r_j – and ending at r_j in this case.

Therefore, this equation well justifies the low use of the Gini difference motivated by the difficulties in its calculation (CHARPENTIER, MUSSARD, and OURAGA, 2021).

$$\Delta = \frac{1}{2} E\left|r_i - r_j\right| \tag{2.1}$$

An alternative for calculating the Gini coefficient is using Equation (2.2). This new equation derives from algebraic manipulation of Equation (2.1) after substitution of $\left|R_i - R_j\right|$ for $r_i + r_j - 2min(r_i, r_j)$ and of the application integration cited in the previous paragraph. In Equation (2.2) r_j represents each realization of the random variable in question and, finally, $F(r_j)$ is the cumulative probability density function of the same variable. The Gini coefficient that Equation (2.2) represents has stood out among the various options tested by researchers as an alternative to the MV model. This model is as intuitive as the variance equation and does not depend on normally distributed data nor a specific form of utility function for its application (BUKOVSEK et al., 2021; MARCONDES et al., 2017).

$$\Delta = 2\, Covar\, [r_j, F(r_j)] \tag{2.2}$$

MG model has other benefits concerning the MV model, such as, for example, its portfolios have average and risk parameters that allow the analysis of necessary conditions of stochastic dominance (SD) in a relatively simple way, if compared to that of the MV model. A very attractive feature, whether in the analysis of financial asset portfolios or in the PPAS (CHARPENTIER, MUSSARD, and OURAGA, 2021).

2.5 Gini Correlation Coefficient

Efficient portfolios, according to MG, can be derived analytically in a similar way to the MV. Then, if the same constraints as the MV model are imposed, these MG portfolios can also be obtained using optimization techniques for constrained minimization problems. However, there is an essential difference between the MG leads concerning the MV, where those from MG are associated with two correlation coefficients between each pair of assets, while those from MV are associated with only one, the well-known Pearson correlation coefficient (DE LA TORRE CRUZ et al., 2020; FURMAN and ZITIKIS, 2017).

In Equation (2.3), we present the two possible correlation coefficients of the MG between two assets i and j are presented, in which r_i and r_j are respectively the values of the random variables of returns of the distributions of assets i and j and, finally, $F(r_i)$ and $F(r_j)$ are the values of the cumulative probability density distribution referring to r_i and a r_j, respectively (YITZHAKI and SCHECHTMAN, 2013).

$$\Gamma_{ij} = \frac{cov[r_i, F(r_j)]}{cov[r_i, F(r_i)]} \qquad \Gamma_{ji} = \frac{cov[r_j, F(r_i)]}{cov[r_j, F(r_j)]} \tag{2.3}$$

When measuring the squared Gini risk of a portfolio Δ_p^2 between two assets i and j according to Equation (2.4), it should be considered that: α_i and α_j are the shares of assets i and j in the portfolio and, by last, Δ_j and Δ_j are the Gini risks referring to the same assets.

2.6 Projects ROI Variations and Covariations

For mapping interdependence between pairs of projects on return on investment (ROI) parameter, we need both Gini correlation coefficients described in Equation (2.3) to decompose individual risk contributions by asset combinations in pairs. It is important to note that the two coefficients are not necessarily equal. They will be equal only if the distributions of i and j are interchangeable in a linear transformation (DE LA TORRE CRUZ et al., 2020).

$$\Delta_p^2 = \alpha_i^2\Delta_i^2 + \alpha_j^2\Delta_j^2 + 2\alpha_i\Delta_i\alpha_j\Delta_j\left[\left(\Gamma_{ij} + \Gamma_{ji}\right)/2\right] \quad (2.4)$$

However, the information that the Gini correlation coefficients Γ_{ij} and Γ_{ji} between pairs of assets are equal or not is left out by practically all researchers of the concept in the scientific community. I.e., all research on this concept is chosen not to consider the sum of the two correlation coefficients in the Gini risk mapping but only is considered Γ_{ij} in the equations. Thus, we can simplify Equation (2.4) to an approximation according to Equation (2.5). In this equation, the risk mapping would be similar to the classic form for variance, if Γ_{ij} represent the Pearson correlation coefficient. However, here the term represents the Gini correlation, as well as the deviation values described by Δ referring to the Gini risk (FURMAN and ZITIKIS, 2017).

$$\Delta_p^2 = \alpha_i^2\Delta_i^2 + \alpha_j^2\Delta_j^2 + 2\alpha_i\Delta_i\alpha_j\Delta_j(\Gamma_{ij}) \quad (2.5)$$

Also, in Equation (2.5), the term $2\alpha_i\Delta_i\alpha_j\Delta_j(\Gamma_{ij})$ infers the correlation between pairs of assets involved. That is, if assets i and j are positively correlated, then the squared risk of the portfolio Δ_p^2 will be greater than the weighted square sum of the risks of the assets that compose it – that is, $\Delta_p^2 > \alpha_i^2\Delta_i^2 + \alpha_j^2\Delta_j^2$. This is due to the increment in the value given by the covariance between the assets – that is, by adding $2\alpha_i\Delta_i\alpha_j\Delta_j\left(\Gamma_{ij}\right)$. Now, if assets i and j are negatively correlated, the squared portfolio risk Δ_p^2 will be reduced. This reduction will occur by subtracting the value of the covariance between the pair of assets i and j – that is, by subtracting $2\alpha_i\Delta_i\alpha_j\Delta_j\left(\Gamma_{ij}\right)$ – from the weighted quadratic sum in question (YITZHAKI and SCHECHTMAN, 2013).

Thus, analyzing Equation (2.5), it is possible to affirm that the benefit of the negative correlation can reduce the Gini risk of the portfolio in question to a value that can reach zero, that is, with $\Delta_p^2 \longrightarrow 0$. This is similar to what demonstrated by Markowitz in 1952 when he mapped the portfolio's variance. This work by Markowitz was later awarded the Nobel Prize in economics (DE LA TORRE CRUZ et al., 2020).

2.7 Portfolios Selection by Maximum Returns per Risk Unit

The model in question, for obvious reasons, assumes that only integer and binary variables are viable, that is, where $x_j \in [0.1]$. This assumption implies significant changes in the ways of solving the adjacent model. The most significant change occurs in the solution viable region, which reduces the horizon of infinite possibilities to a finite and calculable quantity. Therefore, if *S* is the set of finite solutions for all combinations C_{x_j}, thus $C_{x_j} = 1 \dots s$ to $x_j \in [0\ 1]$ (LI et al., 2020; ROLAND, FIGUEIRA and DE SMET, 2016; SHARIATMADARI et al., 2017).

Still, if $j = 1 \dots n$ represents the projects number, then the finite number of combinations for $x_j \in [0\ 1]$ is $p = 2^n - 1$. This considering that not executing any project is not feasible. The problem here is that the adapted mathematical model, with constraints of binary decision variables, prevents using gradient variation algorithms (BUKOVSEK et al., 2021; FURMAN and ZITIKIS, 2017).

However, the model adjusted here does not assume optimized solutions, but there exists a group of solutions $s = 1 \dots p$. That is, for each option of the set, we must calculate the value of SG_S. In turn, SG_S is described in Equation (2.6) with the modification of risk standard deviation by Gini risk. The resulting value of Equation (2.6) index originates from the Sharpe ratio and, at the same time, uses the Gini coefficient as the risk measure. The solution SG_S for each possibility belonging to the set of all feasible combinations of $C_{x_j} \in S = 1 \dots p$ and requires that S be previously defined (LI et al., 2020; KUMAR et al., 2018).

$$SG_S = \left[\left(\sum_{j=1}^{n} r_j\, x_j\right) - R_f\right] . \left\{2\ Covar\ [r_j x_j\ , F(r_j x_j)]\right\}^{-1} \qquad (2.6)$$

Also, all the constraints of the original model no longer make sense because the model does not need optimizations within specified limits. On the other hand, the only restrictions that we must obey in the solution of SG_S are that define the feasible region of the system according to (2.7) and (2.8) (SONG et al., 2019; TAVANA et al., 2020).

The proposal is that after the solution of Equation (2.6) for each possibility belonging to the set of solutions $S = 1 \dots p$ and according to the defined feasibility constraints, then only the best project portfolios should follow as alternatives. That is, only those with the highest index values (BUKOVSEK et al., 2021).

$$C_{x_j} \in [S] \; and \; to \; all \; S = 1 \dots p \tag{2.7}$$

$$x_j \in [0,1] \; and \; to \; all \; j = 1 \dots n \tag{2.8}$$

2.8 Project Portfolios Selection by Beta-Gini Risk

To obtain the beta Gini-CAPM, we must be supposed that portfolio excess returns $\sum_{j=1}^{n} x_j(r_j - R_f)$ are the return excesses of the investor $(R_e - R_f)$. The assumption has conceptual basis on optimal conditions and in performed convenient algebraic for the development. Next, we must consider that the term $1/\lambda_l$ is equivalent to $\sum_{j=1}^{n} x_j(r_j - R_f)/\Delta_e$, in optimal condition, and which thus it can be replaced by $(R_m - R_f)/\Delta_m$. The assumption is according to the CAPM conceptual-basic extended to Gini-CAPM methodology (HOMM and PIGORSCH, 2012).

The development also has based on the existence of risk aversion homogeneity of the scenario participants. The extension of this CAPM premise implies the possibility of changing the individual index j from the optimal portfolio return to the market m index. Thus, a first draft of Gini-CAPM pricing is possible, according to Equation (2.9) (GRIBKOVA and ZITIKIS, 2017).

$$\frac{(R_m - R_f)}{\Delta_m} = \frac{(R_e - R_f)}{\Delta_e} \tag{2.9}$$

The expected relationship between excess returns of an asset or portfolio by excess returns on the market portfolio, according to Equation (2.10), is a classic definition of non-diversifiable beta risk. This definition is represented mathematically on the right side of GB in Equation (2.9) (BUKOVSEK et al., 2020).

$$GB = \frac{\vartheta \Delta_e}{\vartheta x_j} \frac{1}{\Delta_m} \tag{2.10}$$

The beta-Gini can further develop. For this, as already established, it is sufficient to consider that $\vartheta\Delta_e/\vartheta x_j = 0$ has the same value of x_j in $2\ Covar\left[\ r_j x_j, (FC(R_m)\right]$, when $x_j \ \rightarrow x_j{}^*$, exactly at $\vartheta\Delta_e/\vartheta x_j = 0$, i.e., where x_j is optimum. In this way, the formulation $2\ Covar\left[\ r_j x_j{}^*, (FC(R_m)\right]$ represents the best option of excess return per risk among possibilities. Thus, we can write the GB_S Equation (2.11) finally describes the strategic and non-diversifiable beta-Gini risk (GRIBKOVA and ZITIKIS, 2017; HOMM and PIGORSCH, 2012).

$$GB_S = 2\ Covar\left[\ r_j x_j, (FC(R_m)\right]/\Delta_m \tag{2.11}$$

The GB_S is the more critical equation of this Sub-section, and we should apply the equation for every combination $C_{x_j} \in S = 1 \dots p$ and respects the frontiers of the feasible region of the system, according to (2.7) and (2.8). In this case, the research proposal for the application is that, after the solution of GB_S in Equation (2.11), we selected only the portfolios with the best indexes and, according to plan, with values closer to 1 unit. The strategy of looking for values closer to 1 unit "we must adopt when it is not possible to predict the market situation for the investment period" (BUKOVSEK et al., 2021; ISAIAS, PAMPLONA and GOMES, 2015).

2.9 Project Portfolios Selection by Gini Pricing

The project portfolio pricing proposed in this Section, as with CAPM, can be used to provide the rate of return that each asset or portfolio should have. This if they had a ratio of excess return rate per unit of Gini risk equal to the benchmark option. In fact, the plan is to use the Gini-pricing to identify excellent portfolios in forecasting excess return rate per unit of risk (TAVANA et al., 2020). In the development, the first-order optimality condition $\vartheta\Delta_j/\vartheta x_j$ = 0 can be described by Δ_e, when the optimal vector $x_j{}^*$ is known and applied in Equation (2.11). Therefore, it is possible to isolate R_e of Equation (2.11) and to replace the terms corresponding to the same (BUKOVSEK et al., 2021; GRIBKOVA and ZITIKIS, 2017; HOMM and PIGORSCH, 2012).

Also, for every combination $C_{x_j} \in S = 1 \dots p$, R_e in Equation (2.11) must be implemented to calculate the value of each project portfolio. This pricing should use to show opportunities that, in this case, are represented by the most substantial differences between price forecasted and Gini-priced return. Equation (2.12) translates this strategy, and it must be applied to all options concerning the system's feasible region's delimitation constraints to determine the "Gini-price" variable. In the application example of Section 4, portfolios are among the most substantial differences we are selected. Also, it is prudent for this selection line to shrink as the value of $j = 1 \dots n$ increases (FURMAN and ZITIKIS, 2017).

$$R_e = R_f + (R_m - R_f)\{2\ Covar\ \{\ r_j x_j, [F(R_m)]\}/\Delta_m\} \tag{2.12}$$

3 MATERIALS AND METHODS

In this Section are the main developments of this research, in addition to theories strictly linked to the developments, fundamental assumptions, and second-level developments to support the first-level ones. In fact, each subsection will address the following topics: the methodology of this research; the assumptions that are fundamentals to the developed methods; a method to project portfolio selection using criteria Gini-CAPM and Gini-semi variations, which also is analytic to computational cost reduce; a method to project portfolio selection similar to the previous one, but to solar energy by photovoltaic cells, and able to consider interdependence in ROI; and, finally, second-level methods that we developed to enable first-level developments.

3.1 Research Methodology

In practice, the study of research methodology theory should guide the researcher systematically and with a focus on the study design, seeking to ensure valid, reliable results that meet the goals and objectives. For example, methodology definitions should be paramount in deciding: what information and data to collect, what not to collect, who should carry out the collection, how the data will be collected, how to analyze the information and data, among others.

Nevertheless, it is essential to emphasize that the methodology must also explain not only the methodological choices but why we choices them. In other words, the methodology must justify the choices, showing that the methods and techniques chosen are the most suitable for the objectives and goals and, therefore, will provide acceptable results (ROSS and CALL-CUMMINGS, 2020; RYDER et al., 2020). Given the importance of planning methodology and research methods, we recommended that they should be planned at least at a satisfactory level of initial detail. So, to adapt to the expected minimum level, this Sub-section presents the classification for this research, in addition to fundamentals concepts.

The study of the methodology and the definitions of the chosen research methods should guide the execution and, later, help to clarify why we wanted the research, how we carried out, which resources we need for execution, how far the scope is, and others. Thus, the methodology and research methods should help to solve the problem and achieve the objectives in a certain sequence (ROSS and CALL-CUMMINGS, 2020; RYDER et al., 2020).

The sequence for the elaboration of scientific research has approximately the following steps: definition of the theme and sub-themes; problem definition; justifications; elaboration of the thesis; definition of objectives; theoretical foundation; framework in the theory of research methodology; determination of research methods; definitions of forms of data collection and treatment; generations and analysis of results; conclusions on the analysis of results; elaboration and submission of articles; among others (ROSS and CALL-CUMMINGS, 2020).

From the planning of the stages of scientific research, its classification is possible. Furthermore, this is necessary for the researcher to identify the directives to increase the probability of success because, after defining the paths for the research, we will conduct it more easily throughout the process. In fact, in Production Engineering, research is usually: basic or applied (in terms of nature); exploratory, descriptive, explanatory or normative (in terms of objectives); and qualitative, quantitative or combined (in terms of approach). Still, the options of methods in this sense are the experiment, modelling and simulation or survey (when the approach is quantitative); and case study, action research or soft systems methodology (when the approach is qualitative) (RYDER et al., 2020).

As for its nature, basic research seeks scientific progress and the expansion of theoretical knowledge without the concern of using them in practice. Therefore, it is formal research, in view of generalizations, principles, laws and aims at knowledge for knowledge's sake. Applied research, on the other hand, is characterized by its practical interest, where we apply the results or use them immediately in the solution of problems that occur in reality. In fact, basic research would be more linked to the increase of scientific knowledge, without commercial objectives, while applied research would be motivated by commercial objectives through the development of new processes or products oriented to market needs (ROSS and CALL-CUMMINGS, 2020).

As for its objectives, exploratory research aims to provide greater familiarity with the problem to make it explicit or build hypotheses. Still, it involves a bibliographic survey; interviews with people who had practical experiences with the researched problem; analysis of examples that stimulate understanding. Descriptive research, on the other hand, outlines "what it is" and aims to describe the characteristics of a given population or phenomenon or the establishment of relationships between variables. Furthermore, in this case, the option in question involves using standardized data collection techniques: questionnaire and systematic observation. In turn, explanatory research aims to identify the factors that determine or contribute to the occurrence of phenomena and, therefore, deepens the knowledge of reality because it explains the reason, the "why" of things (when carried out in the natural sciences, it requires the use of the experimental method, and in the social sciences it requires the use of the observational method). Finally, in this dimension, normative research is primarily interested in the development of policies, strategies and actions to improve the results available in the existing literature, to find an optimal solution to new problem definitions or to compare various strategies related to a specific problem (ROSS and CALL-CUMMINGS, 2020; RYDER et al., 2020).

As for how to approach the problem, quantitative research considers everything can be quantified, translating opinions and information into numbers to classify and analyze them. It still requires the use of statistical resources and techniques (percentage, mean, standard deviation, correlation coefficient, regression analysis, among others). Qualitative research, on the other hand, considers that there is a dynamic relationship between the real world and the subject, that is, an inseparable link between the objective world and the subject's subjectivity that we cannot translate into numbers. In it, the interpretation of phenomena and the attribution of meanings are basic in the process, and it also does not require the use of statistical methods and techniques. It still assumes that the natural environment is the direct source for data collection, and the researcher is the critical instrument who tend to analyze their data inductively, where the process and its meaning are the main focuses of the approach. Finally, combined research considers that the researcher can combine aspects of qualitative and quantitative research in all or some of the stages of the process (ROSS and CALL-CUMMINGS, 2020).

From the method point of view, we used the experiment when an object of study is determined, we can select the variables that would influence them, and the forms of control and observation of the effects of the variables on the object are knowledge. The survey, on the other hand, should be used when the research involves the direct interrogation of people whose behaviour one wants to know. Modelling and simulation, we can use when we want to experiment, through a model, with a real system, determining how it will respond to the modifications proposed. In turn, the case study involves the deep and exhaustive study of one or a few objects in a way that allows its broad and detailed knowledge. Action research, on the other hand, should be used when conceived the association with an action (or with the resolution of the problem), where researchers and participants are involved in a cooperative or participatory way. Finally, the soft systems methodology should help the formulation and structuring of thinking about problems in complex situations, and its principle is in the construction of conceptual models (based on the understanding of human activities) and in the comparison of these models with the real world (ROSS and CALL-CUMMINGS, 2020; RYDER et al., 2020).

Figure 2 presents the classification for this research: applied in the nature dimension; explanatory as to its objectives; and quantitative in approach. Regarding the research methods, we used modelling and simulation.

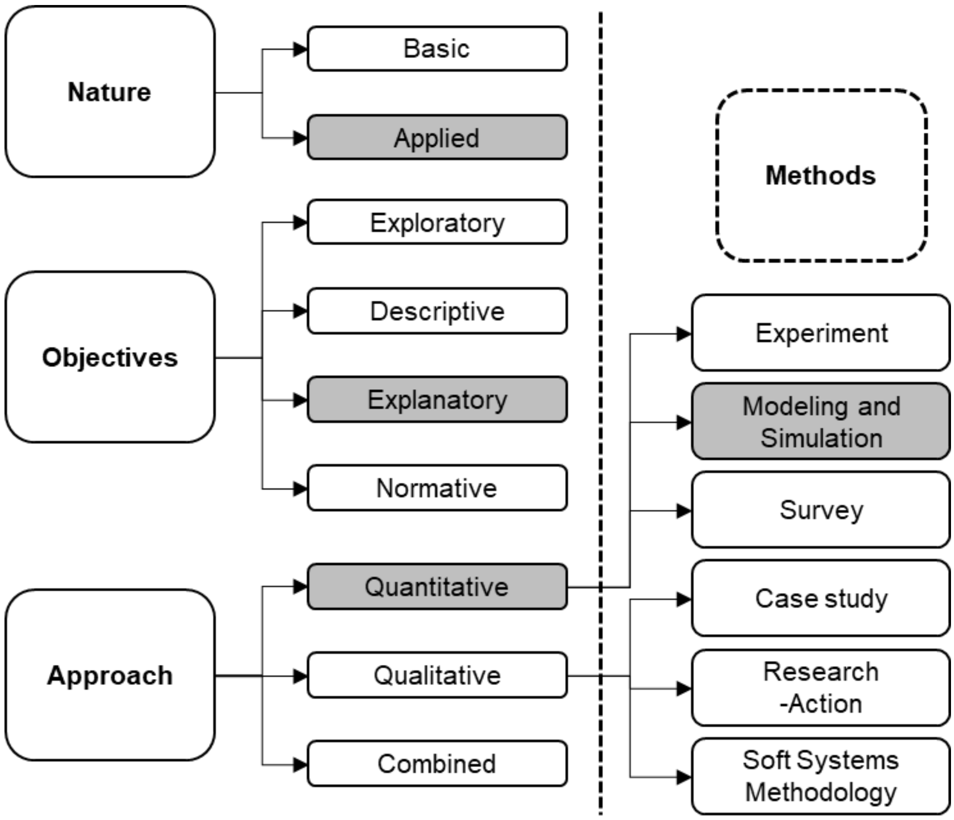

Figure 2 – Classification of scientific research for production engineering
Source: author

3.2 Methods Fundamentals Assumptions

The assumptions presented and discussed here differ in some way from those valid for the CAPM and Gini-CAPM or because they are exclusive to this research. Also, the CAPM and Gini-CAPM assumptions not present in this description we must understand as necessary. That is because the methodologies are the main references for this research. The first assumption discussed comes from the CAPM and the Gini-CAPM: "the data referring to the returns of the assets under study have normal distributions of probability" (AUER and SCHUHMACHER, 2013; MARCONDES, 2016; RINGUEST, GRAVES and, CASE, 2005).

The MV conceptual basis is considered the main barrier to developing a method similar to that of CAPM for applications in PPAS. Therefore, this first assumption is not active for the methods developed by this research, giving them great attractiveness because it represents an opportunity gap. Furthermore, another critical assumption by CAPM is that in it "in the scenario, there are no taxes, regulations or restrictions to short selling" (NHLEKO, and MUSINGWINI, 2016; SHARPE, 1966). This case is not valid for analysing and selecting the project portfolio in the method. More accurately, at least at this first moment, short selling possibilities are not allowed.

Another critical assumption of CAPM and Gini-CAPM says that "the assets are divisible, that is, it is possible to obtain and retain fractions of assets according to the investment strategies adopted" (ASSAF NETO, 2008; GITMAN, 2010). It is not possible to accept that this last assumption is valid for the methods proposed by this research. Among the implications of non-validation, we must highlight the possibility of not meeting optimality conditions according to the reference models of this research, which, in turn, were developed for continuous assets. This last assumption generates some limitations, where the most significant is the imped the use of algorithms based on gradient variation to solutions. The reason is that, if its application were possible, it would be very strategic since these algorithms are very efficient from the point of view of computational exhaustion.

Still, in the initial phase, "the CAPM developers have proven that the returns between assets not only correlate each other but mainly with a single index representative of the market as a whole" (ASSAF NETO, 2008). The index mentioned above is of paramount importance for this research. In defining the market portfolio, in CAPM and Gini-CAPM models, it is possible to verify the recommendations is that the ideal is to maximize the excess return per risk unit concerning a risk-free rate R_f. Furthermore, this rate value and the information related to the scenario must be available to all investment participants (MALLER, ROBERTS, and TOURKY, 2016; SHARPE, 1963).

Thus, if r_j is the expected return on the asset or portfolio j financial or of projects and, R_f is the risk-free rate available for the scenario, then a rational investor will only invest in j if $r_j - R_f > 0$. That is, the investor should only invest in the asset or portfolio j, if there is a prediction of a positive difference between in r_j and R_f. And even more importantly, the cited difference must satisfy the investor's requirements to a level to conclude that he is worth the forecast's risk not being realized. The difference mentioned is called the "investor's risk premium" (KOURTIS, 2016).

The essence in defining the market portfolio at CAPM (and Gini-CAPM) is to consider it more important to look for excess returns concerning a reference instead of focusing only on maximum return or minimum risk. So, in the development of CAPM, Sharpe proposed that the best investment option should be the one that presents the mentioned best excesses per unit of risk from the R_f. The return index of the market portfolio at CAPM is the Sharpe Ratio, wherein the literature it is common to describe him as the excess return per unit of total risk. Also, another commonly found literary description is that the index is the metric of risk units, that the return $\sum_{j=1}^{n} r_j\, x_j$ may fall until losses occur, i.e., until the return is below the R_f. The portfolio's suitability to the investor, on the other hand, must occur due to its risk tolerance (MALLER, ROBERTS, and TOURKY, 2016; SHARPE, 1963).

After all the theoretical bases, it is possible to affirm that the market portfolio here, in the "Gini-CAPM for Projects", can have three estimation forms. In decreasing order according to the arbitrated ideal, the first is the index that best represents the market that the organization houses. In sequence, the portfolio index with maximum excess returns per Gini unit within the organization's possibilities. Finally, the index of the portfolio with maximum excess returns per Gini unit but discreet and doable and within the organization's prospects.

Thus, due to its representativeness, the ideal is to define the portfolio index discussed here to represent the organization's market best. For this, it must assume that the implementer of any of the methods can define this index. In turn, for example, by estimates of the mean return of sector organizations in the country in which it operates; by estimates of the rate of return of groups in the same sector on the most representative stock exchange; by to the minimum attractiveness rate determined internally following organizational policies, and others. However, this way defined as ideal for estimating the market index induces another even more critical estimate. I.e., any organization that implements the method should establish the Gini correlation between each project's market index or of each possible pair of projects. This Gini correlation parameter is necessary for calculating the non-diversifiable risk Beta-Gini and other adjacent parameters.

It is worth mentioning that both the market index and the definition of the critical levels of correlation presuppose excellence in financial management. According to the PMI (2017), for established methodologies of estimates are good options the parametric, analogues, specialists, bottom-up, Pert, and others. Another possibility, the second option, which gives up some of the market index's representativeness, is to consider the global optimum in excess returns per Gini risk unit, wherein project scenario, the optimum portfolio is the "index Sharpe-Gini." Suppose the set of projects under analysis is large enough to represent, on average, the projects commonly executed in the organization. In that case, it is possible to assume it as a reasonable representation of the market.

The second alternative to estimate the market index should occur if the first is impossible. This statement has based on the "fundamental differences in dynamics between a portfolio of financial assets and projects" (BREALEY, MYERS, and ALLEM, 2013; PMI, 2017). The third alternative presented to estimate the market index also generates a less representative result than the second. In this case, the index is the discrete optimum in excess returns per unit of Gini risk. The main advantage will be that the resulting portfolio is entirely doable. This index will also gain representativeness if the set of projects under analysis is quite broad.

In fact, in the application examples studied in Section 4, the second method indicated is used to estimate the market index. This form has a slight advantage over the first in obtaining Gini correlations, together with the third, because the correlations are calculated based on the projects' presence in the market portfolio. It is essential to clarify that this last assumption focuses on another great relevance for the research. The applications of the methods discarded all the composition strategies with the optimal market portfolio. Furthermore, this applies to all possible operations: short, long, leveraged, unleveraged, hybrid, and others.

In the scenario, we assume that the portfolio is similar to the market optimal, predictable, but not differentiable. This assumption certainly increases the scientific community's acceptability by imposing a more realistic structure. However, the theme "market portfolio index and the Gini correlations estimation of each project towards the same" is paramount in this research. It is also possible to affirm that this is of high complexity to justifying exclusive research.

3.3 PPAS using CAPM, Semi-Variations, and Gini Risk

In this Subsection, we present the first method resulting from this research after exhaustive tests and adjustments. This method selects project portfolios using the Gini-CAPM criteria (maximums excess return per unit of risk, and minimums non-diversifiable beta-Gini risk) and semi variations (skewness and stochastic dominance). In fact, in the results and discussions about this method presented in Subsection 4.1, we apply it with a technological increment to reduce, according to computational cost presented in Subsection 3.5.1.

An initial definition needed is the risk-free rate R_f, which represents a scalar, and generally, its adopted value corresponds to national treasury bills. In practice, R_f means the minimum level of profitability without risk to a country, but also it is possible to implement organizational particularities such as rates of attractiveness minimum, among others. Another critical input parameter is the ROI that each project will offer. With the ROI values, we should generate a matrix R_{ij}, where $i = 1 \dots m$ represents the parameters of the distributions of the project $j = 1 \dots n$. In the matrix R_{ij} each term t in dimension i represents, respectively, the minimum, the most probable, and the maximum value of the triangular probability distributions for the project j. These values we must estimate according to organizational convenience (for example, using the Program Evaluation and Review Technique - PERT) (PMI, 2017).

$$R_{ij} = \begin{bmatrix} t_{11} & \dots & t_{1n} \\ \dots & \dots & \dots \\ t_{31} & \dots & t_{3n} \end{bmatrix} \tag{3.1}$$

And with the information available so far, it is possible to calculate the individual Gini risk Δ_j of each project. It is essential to highlight the contribution in Section 3.1 to calculate Gini risk because the development presented in it allows reductions in the application's cost and time. According to Section 3.4.1, if the parametric analytical result of the Gini risk coefficient Δ_j of the triangular distribution has their limits in a and b, and the most likely value in c, then Equation (2.2) can measure the Gini risk Δ_j of each project $j = 1 \dots n$.

We describe Equation (3.2) the Gini square Δ_{jj}^2. In the equation, it is essential to highlight the $2\alpha_j\Delta_j\alpha_{j'}\Delta_{j'}(\Gamma_{jj'})$ means that if assets j and j' are positively correlated, then the square risk of the portfolio $\Delta_{jj'}^2$ will be greater than the weighted quadratic sum of the risks of the assets by α_j and $\alpha_{j'}$. And if the assets j and j' are negatively correlated, the square risk of the portfolio will reduce.

$$\Delta_{jj'}^2 = \alpha_j^2\Delta_j^2 + \alpha_{j'}^2\Delta_{j'}^2 + 2\alpha_j\Delta_j\alpha_{j'}\Delta_{j'}(\Gamma_{jj'}) \tag{3.2}$$

Now we should calculate the optimal market portfolio GS_s^* similar to that of Sharpe from Gini-CAPM. This portfolio is not differentiable as its calculation does not have the constraint of $x_j \in [0\ 1]$ for all $j = 1 \ldots n$. The set of equations from Equation (3.3) to Equation (3.4) are necessary and sufficient to determine the optimal portfolio GS_s^*. In set we have z_j, which is a vector of maximum participation of the projects concerning the microenterprise's resources, $3^{(1-\sum_{j=1}^{n} x_j)}$, which represents an empirical mapping of the increase in R_f as the company's debt increases. In the case of $3^{(1-\sum_{j=1}^{n} x_j)}$, if $\sum_{j=1}^{n} x_j > 1$ it must be active, otherwise $3^{(1-\sum_{j=1}^{n} x_j)} = 1$.

The optimal portfolio with x_j^* according to the model of Equation (3.3) to Equation (3.4) will is necessary to calculate the non-diversifiable Gini risk. Also, according to Gini-CAPM, the x_j^* presents the maximum excess of return per unit of risk, the maximum deviation from the mean in units of risk to R_f, the minimum probability of returns less than R_f, among others. In sequence, now we must elaborate the boundary of risk and return, which is similar to Sharpe from Gini-CAPM, and its graphic could make the application more intuitive. We used this boundary in the application of Section 4.1, where we can see that it is continuous and encompasses the market portfolio optimal.

$$\text{Max} \quad GS_s^* = \left[\left(\sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}\, x_j z_j\right) - R_f . 3^{(1-\sum_{j=1}^{n} x_j)}\right] . \left\{2\; Covar \left[(r_{ij}x_j z_{j\_} R_f . 3^{(1-\sum_{j=1}^{n} x_j)}\;, F\; \left(r_{ij}x_j z_j - R_f . 3^{(1-\sum_{j=1}^{n} x_j)}\right)\right]\right\}^{-1} \tag{3.3}$$

$$\text{s.t.} \qquad 0 \le x_j z_j \le z_j \tag{3.4}$$

Finally, we arrived at the first selection criterion, where we must apply Equation (3.5) under the restrictions (3.6) and Equation (3.7). We adapted the set from Equation (2.6) under the restrictions (2.7) and Equation (2.8) due to the discrete scenario of projects, and for R_f = R_f until $\sum_{j=1}^{n} x_j \le 1$, and to the resultant of the product by $3^{(\sum_{j=1}^{n} x_j - 1)}$ to $\sum_{j=1}^{n} x_j > 1$. The value of Equation (3.6) is the index GS_s, derives from the Sharpe index, and uses the Gini coefficient as the risk metric. The portfolios $x_j^* \in [0\ 1]$ in this case is differentiable.

$$SG_S = \left[\left(\sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}\, x_j z_j\right) - R_f . 3^{(\sum_{j=1}^{n} x_j - 1)}\right] . \left\{2\; Covar \left[(r_{ij}x_j z_{j\_} R_f . 3^{(\sum_{j=1}^{n} x_j - 1)}\;, F\; \left(r_{ij}x_j z_j - R_f . 3^{(\sum_{j=1}^{n} x_j - 1)}\right)\right]\right\}^{-1} \tag{3.5}$$

$$to\ C_s\ and\ to\ all\ s = 1 \ldots p \tag{3.6}$$

$$to\ x_j \in [0\ 1]\ and\ to\ all\ j = 1 \ldots n \tag{3.7}$$

At this moment, we will also have the necessary information to calculate the non-diversifiable risk values of the portfolios, which we will use as the second selection criterion. Equation (2.11), restricted by (2.7) and (2.8) in an adaptation similar to the previous ones, allows the development of Equation (3.8), which also we must restrict then by (3.6) and by (3.7). As the name implies, this index indicates the risk that we cannot eliminate by diversification. Nevertheless, on the other hand, knowing their values is strategic.

$$GB_S = 2\ Covar\ [(r_{ij}x_jz_{j-}R_f.3^{(\sum_{j=1}^{n}x_j-1)}, F\ (r_{ij}x_jz_j - R_f.3^{(\sum_{j=1}^{n}x_j-1)})/\Delta_m \quad (3.8)$$

Again, aiming to make the method more intuitive and understandable, a graphical representation is strategic at this moment. We used this graph in the Section 4.1 application, where we represented the ROI x non-diversifiable risk variation for all differentiable options. Furthermore, at this point, we must calculate the skewness of each portfolio option within the feasible set $s = 1 \dots p$.

Reminiscing, skewness comes from the moments of a function, which are quantitative measures related to the shape of its graph, and which the concept is closely related to the moment in physics. However, here, we use its extension to probabilities distributions functions. In this case, the first moment is the expected value, the second moment is the variance, and the third is the skewness. Equation (3.9) presents the generalization of the concept, where n represents the moment in question.

$$\mu^n = \int_{-\infty}^{\infty} x^n\ f(x)dx \quad (3.9)$$

Equation (3.10) presents the solution of Equation (3.9) concerning the third moment. However, it is essential to note a contribution in Equation (3.10), where the risk metric is the Gini coefficient, and the calculations are analytics using the development presented in Section 4.1. The equation solution also uses Pearson's first approximation, where the main references are the portfolio mean $\bar{x}_s$, the portfolio mode $\hat{x}_s$, and in the case of this adaptation, the Gini risk Δ_j. The results of Equation (3.10) are the base values for the method's third selection criterion.

$$\mu_S^3 = \frac{E[(X-\mu)^3]}{(E[(X-\mu)^2]^{3/2}} = (\bar{x}_s - \hat{x}_s)\left[\sum_{j=1}^{n}\frac{2a_j^2 + 2b_j^2 + c_j^2 - 3a_jb_j - a_jc_j - b_jc_j}{15\left(b_j - c_j\right)}\right]^{-1/2} \quad (3.10)$$

In the sequence, we arrive at the fourth criterion for selecting the method, the stochastic dominance of Gini. In fact, in comparisons between portfolios, this characteristic is quite strategic, as it indicates greater probabilities of desired returns or of minimal risk. Inequality (3.11) presents the first condition of stochastic dominance, which is a simple comparison of mean $\sum_{j=1}^{n} r_j\, x_j$ of two portfolios. Subsequently, if and only if (3.11) is true, then (3.12) is needed to establish a second-order stochastic dominance in the comparison.

Equation (3.12) contributes by extending the development presented to Gini's second-order stochastic dominance. In this case, if both conditions exist, we can say that, at a second-order level, the portfolio j dominates stochastically the portfolio j'. It is also important to emphasize that if only one of the inequalities exists, the other can be an equality. Also, the stochastic dominance will be fully guaranteed only if the graph of the probability density functions in the comparison has only one intersection at most.

$$\sum_{j=1}^{n} r_j\, x_j \;\geq\; \sum_{j'=1}^{n} r_{j'}\, x_{j'} \tag{3.11}$$

$$\sum_{j=1}^{n} r_j\, x_j - \left[\sum_{j=1}^{n} \frac{2a_j^2 + 2b_j^2 + c_j^2 - 3a_jb_j - a_jc_j - b_jc_j}{15\,(b_j - c_j)}\right]^{1/2} \geq \sum_{j'=1}^{n} r_{j'}\, x_{j'} - \left[\sum_{j'=1}^{n} \frac{2a_{j'}^2 + 2b_{j'}^2 + c_{j'}^2 - 3a_{j'}b_{j'} - a_{j'}c_{j'} - b_{j'}c_{j'}}{15\,(b_{j'} - c_{j'})}\right]^{1/2} \tag{3.12}$$

The last two criteria included in this modelling have base on metrics on semi-variations. Furthermore, a very close graphical representation of these two criteria is the feasible efficient frontier of the scenario. Thus, to increase the understanding of the method application, the definition of the boundary in question is now strategic. Finally, we use a tool to weigh the criteria, classify the investment options, and highlight the best opportunities. We use Analytic Hierarchy Process (AHP), a multicriteria decision method based on hierarchical structuring. The AHP choice is because “it is the most applied in decisions based on multiple complex criteria” (LOUREIRO, GOLDMAN and NETO, 2018). The pseudocode in Figure 3 summarizes the steps of the method that we detailed in this Section.

BEGIN
$R_f \leftarrow load\ the\ risk - free\ rate\ scalar$
$R_{3j} \leftarrow load\ the\ ROI\ matrix\ of\ all\ projects$
$\Delta_s \leftarrow analytically\ calculate\ the\ Gini\ risks\ of\ projects\ and\ portfolios$
FOR $j = 1{:}n$
$GS_s^* \leftarrow calculate\ the\ optimal\ portfolio\ similar\ to\ of\ Sharpe$
END FOR
$GP_1 \leftarrow generates\ the\ boundaries\ graph\ of\ returns\ per\ risk\ Gini$
FOR $s = 1{:}p$
$GS_s \leftarrow calculate\ all\ discret\ portfolios\ excess\ ROI\ per\ Gini\ risk$ unit
END FOR
FOR $s = 1{:}p$
$GB_s \leftarrow calculate\ all\ the\ portfolios\ beta\ Gini - CAPM$
END FOR
FOR $s = 1{:}p$
$\mu_s^3 \leftarrow calculate\ all\ the\ portfolios\ Gini - Skewness$
END FOR
FOR $s = 1{:}p$
$SD_s \leftarrow calculate\ all\ the\ portfolios\ Gini - Stochastic\ Dominance$
END FOR
$GP_2 \leftarrow generates\ the\ graph\ similar\ to\ securities\ market\ line$
$Rank \leftarrow apply\ the\ AHP\ model\ on\ the\ GB_s, GB_s, \mu_s^3\ and\ SD_s$
$Selection \leftarrow select\ the\ best\ portfolios$
END

Figure 3 – Pseudocode of the Method I

Source: authors

3.4 PPAS Gini-CAPM, Gini-Semi-Variations and Considering CoGini

In this Subsection, we present the second method resulting from this research, which is an evolution of the first. That is, this method also selects project portfolios using Gini-CAPM criteria (maximum excess return per unit of risk and minimum non-diversifiable beta-Gini risk), but their third criterion is Gini pricing. Nevertheless, in this case, we have two significant contributions: the method has a structure to consider the interdependence of ROI (according to the theory of Section 2.6), in addition to having been designed for specific application in projects of photovoltaic energy generation by solar cells.

CAPM is a methodology that can indicate an optimal portfolio in excess returns per unit of deviation risk, which is the one with the highest Sharpe ratio. This portfolio is supposed to the more representative in the financial securities scenario, and it is the best estimate of the market. Therefore, this research has this portfolio as the market index, as an example of the application of this method in Section 4.2. The premise implies another of great importance for the method, which assumes that the portfolio similar to the market is optimal but not differentiable.

Despite having a new proposal, the method which our research presents are simple. The study proposes a project portfolio select method of SEPC using multi-criteria similar to the Gini-CAPM. The aim is to propose a structured way for small and micro investors to choose SEPC project portfolios. Also, due to the legislation's incentives, the scope of dimensions limits is for small and micro-companies. The reason is that national legislation allows more advantageous uses of photovoltaic energy in this companies’ dimension.

However, changes in solar cell parameters in the application environment are significant, mainly due to heat and humidity. Therefore, it would be necessary several tests to verify the absolute reliability and durability of the equipment according to their respective installation locations. A significant factor that we must consider is the temperature, which usually varies a lot between periods of day and night. Thus, comparative studies are essential to determine the actual efficiency, as well as the useful life of photovoltaic cell systems. The ageing of photovoltaic modules due to high levels of heat and humidity is also very strategic in considering the life and efficiency of the modules.

However, another initial definition of data to the method application is the risk-free rate R_f. This rate represents a scalar value, and generally, its adopted value corresponds to national treasury bills. In practice, it means the minimum level of profitability without risk to a country. However, also it is possible to follow organizational particularities such as minimum rates of attractiveness, among others. Another critical input parameter is the ROI that each project will offer. For this, we need dates to calculate the cash flows of all projects according and then generate a matrix R_{ij}, where $i = 1 \ldots m$ represents the distribution parameters of each project from $j = 1 \ldots n$.

Precisely the same as presented in the first method, in the matrix R_{ij} each term t in dimension i represents, respectively, the minimum, the most probable, and the maximum value of the triangular probability distributions for each project j. The values we must estimate according to organizational convenience, for example, using the Program Evaluation and Review Technique (PERT). The PERT t also receives the name of "Three-Point Estimation" and is the more recommended by the ©PMBOK bestseller (PMI, 2017).

$$R_{ij} = \begin{bmatrix} t_{11} & \cdots & t_{1n} \\ \cdots & \cdots & \cdots \\ t_{31} & \cdots & t_{3n} \end{bmatrix} \tag{3.13}$$

At this moment, the method should establish the Gini correlation matrix $\Gamma_{jj'}$. The matrix is necessary to calculate the Gini's covariations between projects, the Beta-Gini, and the Gini price. “There are other established project estimation methodologies, according to PMI (2017), such as the parametric, or any other, as appropriate (analogue, by the experts, bottom-up, PERT, among others”. We can describe the values of the random return variables of the distributions of assets j and $j\,'$ as r_j and a $r_{j'}$; its two possible Gini correlation coefficients between the assets as $\Gamma_{jj'}$ and $\Gamma_{j'j}$ and, finally, $F(r_j)$ and $F(r_{j'})$ as the values of the cumulative probabilities densities distributions for r_i and a r_j, strictly according to Equation (2.3).

And with the information available, it is possible to calculate the individual Gini risk Δ_j of each asset, and the contributions of that risk by forming portfolios mapped in pairs Δ_{jj} concerning ROI. We can describe the ROI variations and covariation by $\sqrt{\Delta_S^2}$. Still, Equation (2.5) presents the formulation for Gini risk calculating intuitively. Also intuitive is the formulation for the individual calculation of the Gini risk in Equation (3.14), which, in fact, is identical to Equation (2.2).

$$\Delta_j = 2Covar[x_j, F(x_j)] \tag{3.14}$$

It is important to emphasize that, from this point on in this Section, the models presented receive adaptations for application in the scenario (PPAS). According to research in the Web of Science and the Scopus database, the transformations represent theoretical contributions.

Now we should calculate the optimal market portfolio GS_S^* (where * represents optimum) similar to that of Sharpe from Gini-CAPM. This portfolio is not differentiable as its calculation does not have the constraint of $x_j \in [0\ 1]$ for all $j = 1 \ldots n$. The set of equations from Equation (3.15) to Equation (3.19) are necessary and sufficient to determine the optimal portfolio GS_S^*. In Equation (3.15) to Equation (3.19), we have not yet described L, which represents the desired lower ROI limit, $P_{ij}x_j$, which is the investment cost value of each project segmented by periods i, B represents the upper limit of total spending on investments, and z_j is respective the vector of maximum investment values.

$$\text{Max} \quad GS_s^* = \left[\left(\sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}\, x_j\, z_j\right) - r_f\right].\left\{2\; Covar\; [r_{ij}x_jz_j, F\;(r_{ij}x_jz_j)]\right\}^{-1} \qquad (3.15)$$

$$\text{s.t.} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}\, x_j z_j \geq \; L \qquad (3.16)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} P_{ij}\, x_j z_j \leq \; B \qquad (3.17)$$

$$\sum_{j=1}^{n} x_j z_j \leq 1 \qquad (3.18)$$

$$0 \leq x_j \leq 1 \qquad (3.19)$$

The optimal portfolio with x_j^* according to model Equation (3.15) to Equation (3.19) is necessary to calculate the non-diversifiable Gini risk and the Gini price. Also, according to CAPM and Gini-CAPM x_j^* presents the maximum excess of return per unit of risk, the maximum deviation from the mean in units of risk to R_f, the minimum probability of returns less than R_f, among others (RAO, 2009).

In the method, now we can elaborate on the boundaries of risk and return. These boundaries are similar to Sharpe from Gini-CAPM and could make the application more intuitive. We used these boundaries in the application of Section 4.2. The boundaries are: the continuous and necessary for determination the similar to market portfolio optimal; the discrete and real or feasible; and the similar to the market line by R_f compositions.

At that point in the method, we arrived at the first selection criterion. For this, we must apply Equation (3.20) under the restrictions of the expression (3.21) and (3.22), where_the model does not assume optimized solutions but all possibilities. The value resulting from Equation (3.20) is the index GS_s, derives from the Sharpe index and uses the Gini coefficient as a risk metric. The portfolios optimum $x_j^* \in [0\ 1]$ in this case is differentiable. Then, it is a real best option in excess of return per unit of risk, in deviation from the average in units of risk, in the minimum probability of undesirable returns, among others.

$$GS_S = \left[\left(\sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}\, x_j z_j\right) - R_f\right] . \left\{2\ Covar\ [r_{ij} x_j z_j\ , F\ (r_{ij} x_{j z_j})]\right\}^{-1} \quad (3.20)$$

$$x_j \in [0\ 1]\quad for\ all\ j\ =\ 1 \dots n \quad (3.21)$$

$$0 \leq z_j \leq 1 \quad for\ all\ s\ =\ 1 \dots n \quad (3.22)$$

Following the method, we also will have the necessary information to calculate the non-diversifiable risk values of the portfolios to use as a second selection criterion. We can use the non-diversifiable Gini-risk to have accurate decision-making. A classic definition of non-diversifiable risk Beta is the expected relationship between excess returns for a given portfolio and excess returns for the market portfolio.

Equation (3.23) presents the Beta-Gini non-diversifiable risk. As the name implies, this index is where it is not possible to eliminate it through diversification. Nevertheless, on the other hand, knowing values and learning how to manipulate them is strategic. Equation (3.23) must be applied considering $j = 1 \dots n$ and $x_j \in [0\ 1]$, for all $C_{sj} \in s = 1 \dots p$, and $p = 2^n - 1$, and under the restrictions of the expression (3.21) and (3.22).

$$BG_S = 2\ Covar\left[\, r_{ij} x_j z_j, (FC(R_m)\right]/\Delta_m \quad (3.23)$$

Aiming to make the method more intuitive and understandable, the similar graphical representation of the market line is strategic at this moment. We used this graph in the application seen in Section 4.2. Thus, we can represent the variation of ROI x non-diversifiable risk for all the differentiable portfolio options. Over time we hope to identify that assets to move in the same direction in the capital markets, albeit in different proportions. For example, assets try to go up when the market goes up, but according to their Beta risk. Therefore, the non-diversifiable risk is information essential and strategic.

Now we must calculate Gini's pricing values. These are the third selection criterion. According to Gini-CAPM, this pricing can indicate the return rate according to the no-diversifiable risk of an asset. Moreover, the rate, compared to what the asset can offer, can indicate opportunities. Equation (3.24) presents the formula for Gini-CAPM pricing, and again we must apply the equation under the restriction's expressions (3.21) and (3.22).

$$GP_s = R_f + (R_m - R_f)\{2\ Covar\ \{ r_{ij}x_jz_j, [F(R_m)]\}/\Delta_m\} \qquad (3.24)$$

Finally, again it is now necessary to use a tool to weigh the criteria, classify the investment options, and highlight the best opportunities. For this purpose, we use Analytic Hierarchy Process (AHP), which is a multi-criteria decision method based on hierarchical structuring. With the AHP, we can structure the problem using criteria to evaluate, and for this, we must use hierarchical diagrams structured to compare.

The pseudocode in Figure 4 summarizes the steps of the method that we detailed in that Section.

BEGIN
  $R_f \leftarrow load\ the\ risk - free\ rate\ scalar$
  $R_{3j} \leftarrow load\ the\ ROI\ matrix\ of\ all\ projects$
  FOR $j = 1{:}n,\ j' = 1{:}m$
    $\Gamma_{jj'} \leftarrow calculate\ the\ Gini\ correlation\ matrix$
  END FOR
  IF $j = j'$
    $\Delta_{jj'} = [\Delta_j]^2 \leftarrow calculate\ all\ the\ projects\ Gini\ risk\ individual$
  ELSE $j \neq j'$
    $\Delta_{jj'} = [\Delta_{jj'}]^2 \leftarrow calculate\ the\ Gini - covariations\ to\ all\ projects\ pairs$
  END IF
  FOR $j = 1{:}n$
    $GS_s^* \leftarrow calculate\ the\ optimal\ portfolio\ similar\ to\ of\ Sharpe$
  END FOR
  $GP_1 \leftarrow generates\ the\ boundaries\ graph\ of\ returns\ per\ risk\ Gini$
  FOR $s = 1{:}p$
    $GS_s \leftarrow calculate\ all\ discret\ portfolios\ excess\ ROI\ per\ Gini\ risk$ unit
  END FOR
  FOR $s = 1{:}p$
    $GB_s \leftarrow calculate\ all\ the\ portfolios\ the\ beta\ Gini - CAPM$
  END FOR
  FOR $s = 1{:}p$
    $GP_s \leftarrow calculate\ all\ the\ portfolios\ princing\ Gini - CAPM$
  END FOR
  $Rank \leftarrow apply\ the\ AHP\ model\ on\ the\ GB_s, GB_s\ and\ GR_s$
  $Selection \leftarrow select\ the\ best\ portfolios$
  $GP_2 \leftarrow generates\ the\ graph\ similar\ to\ securities\ market\ line$
END

Figure 4 – Pseudocode of the Method II

Source: authors

3.5 Methods for Increment Technological and Support

Here we present two contributions, where the first is a significant technological increase, and the second is an exclusive development to support the first level developments. The first is the analytical algebraic development of formulations to calculate the Gini risk through the parameters of the distributions used in this research. The second method generates distributions correlated by the Gini coefficient at controlled levels.

3.5.1 Analytical Formulations for Triangular Distributions

With the development proposed here, it will be possible to eliminate the need for simulations in the method applications and minimize costs and a literary gap. Moreover, the simulations are not trivial for all the community and generally incur a long time. For example, in the case study presented in Section 4, the step with modelling and simulation for the risk Gini calculation would last at least 6 hours. However, with the help of development, time decreased to 30 minutes approximately.

In addition to the algebra itself, we performed many tests to prove that the development presented here is correct. In these tests case, we observed that the value found with the formulation is always higher and depends on the amount of simulated data (examples: 2.000 data ≅ 9% bigger; 50.000 data ≅ 4% bigger and; 1.000.000 data ≅ 2% bigger). Therefore, the tests lead us to believe that we can generalize due to the simulations just approaching the limits of the distributions. I.e., when the amount of simulated data tends to infinity, the difference will tend to zero.

Starting with algebraic development, according to Charpentier, Mussard and Ouraga (2021); Marcondes et al. (2017), the primary development formula for Gini risk Δ_j of a triangular distribution of r_j is described according to Equation (3.25), where $F(r_j)$ is the cumulative function of the variable r_j.

$$\Delta_j = 2\ Covar\left[r_j, F\left(r_j\right)\right] \tag{3.25}$$

The great question here is to define the covariance of Equation (3.25) in a parametric structure from descriptions of the triangular function that explains r_j. Looking for a more intuitive way, the nomenclature of the term $Covar\ [r_j, F(r_j)]$ change to $Covar\ (x, y)$, more just for the sake of terminology. In the sequence, the opening allows the description according to Equation (3.26):

$$Covar\ (x, y) = E(xy) - E(x)E(y) \tag{3.26}$$

We can calculate the expected value by definite integral, and we can rewrite Equation (3.26) according to Equation (3.27). Also, it is possible to observe at the beginning of the development which we already defined the limits of integrating the two functions. As $f(x)$ represents a triangular, it had its extremes described by parameters a and b, and the most probable result as c, respective to the start, end, and mode values of any triangular distribution, according to Figure 5. We must also consider that $f(y)$ has limits between 0 and 1 because it is a cumulative probability density function of x.

$$Cov(xy) = \int_0^1 \int_a^b x\, y\, f(x,y) dx\, dy - \int_a^b x\, f(x) dx \int_0^1 y\, f(y) dy \tag{3.27}$$

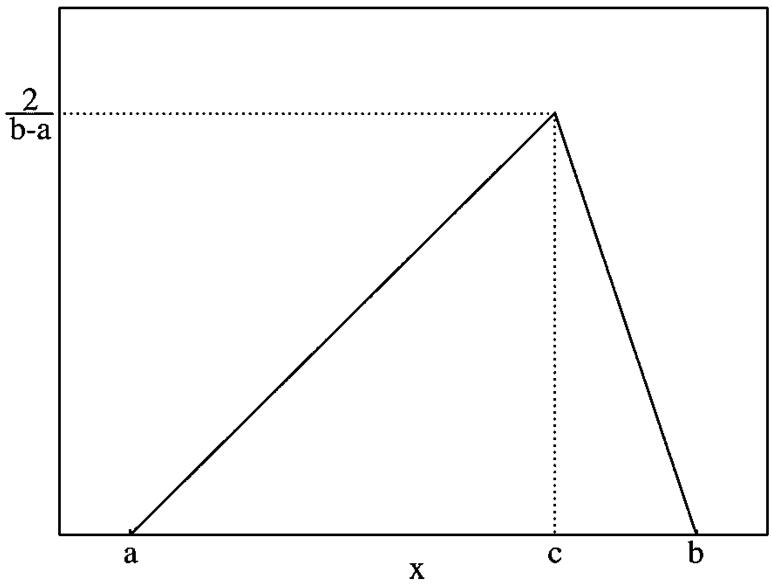

Figure 5 – Triangular distribution parameters

Source: authors

Without going into the merits of the difficulties of solving Equation (3.27) using bivariate functions concepts, another much easier way exists because x and y are correlated "to a high degree." In fact, the most critical part of this development was understanding that y is very dependent on x, so much so that even we can write y function with x parameters. This understanding finally allowed us to arrive at the parametric result of the Gini risk coefficient of a triangular distribution with the algebraic calculus of Equation (3.27). We solved the equation according to the guidelines described in this Section, as shown in Equation (3.28), and we were always using the ©Python software.

The algebraic manipulations to calculate the resulting formula in this Section are simple, but, in fact, we omitted some steps. In other words, only the beginning and the end of development are available here. However, anyone can request the complete development at any time.

3.5.2 Generation of Triangular Distributions with Gini Correlation

As is typical, during the research, to enable first-level, some second-level developments became necessary. This is because we did not find this second-level development in the literature or when we found they assumed extensive needs. In this sense, the development present here stands out to simulate joint triangular distributions and correlated by the Gini parameter. More specifically, with two distributions to describe the assets i and j, then, different from Pearson's correlation, the Gini correlation will present two distinct values Γ_{ij} and Γ_{ji}.

According to the measurement direction, the values of Γ_{ij} and Γ_{ji} are obtained, where it is essential to note that the two coefficients are not necessarily different. Furthermore, in fact, they will be the same if the distributions of i and j are interchangeable in a linear transformation (SHALIT, and YITZHAKI, 1984; YITZHAKI, and SCHECHTMAN, 2013). After researching interchanges of distributions in linear transformations, it was possible to conclude that their concept could simulate correlated distributions. Furthermore, the transformation gains even more appeal if systematized for triangular distributions, which we used in this research, and if a form for unprecedented.

$$\Delta_j = \int_0^1 \int_a^b x\, y\, f(x,y) dx\, dy - \int_a^b x\, f(x) dx \int_0^1 y\, f(y) dy$$

$$\Delta_j = \left[\int_a^c x\, x_{\to y}\, f(x) dx\right] + \left[\int_c^b x\, x_{\to y}\, f(x) dx\right]$$
$$- \left[\int_a^c x. \frac{(2x-2a)}{(b-a)(c-a)}\, dx\right.$$
$$\left. + \int_c^b x. \frac{(2b-2x)}{(b-a)(b-c)}\, dx\right] \left[\int_0^{p\left(x \le \frac{b-a}{2}\right)} y. \frac{(2y-2.0)}{(1-0)(1/2-0)}\, dy\right.$$
$$\left. + \int_{p\left(x \le \frac{b-a}{2}\right)}^1 y. \frac{(2.1-2y)}{(1-0)(1-1/2)}\, dy\right]$$

$$\Delta_j = \left(\int_a^c x \,. \left[\frac{(x-a)^2}{(b-a)(c-a)}\right] . \left[\frac{2(x-a)}{(b-a)(c-a)}\right] dx\right.$$
$$\left. + \int_c^b x \,. \left[1 - \frac{(b-x)^2}{(b-a)(b-c)}\right] . \left[\frac{2(b-x)}{(b-a)(b-c)}\right] dx \right)$$
$$- \left[\int_a^c x. \frac{(2x-2a)}{(b-a)(c-a)}\, dx\right.$$
$$\left. + \int_c^b x. \frac{(2b-2x)}{(b-a)(b-c)}\, dx\right] \left[\int_0^{\frac{1}{2}} y. \frac{(2y-2.0)}{(1-0)(1/2-0)}\, dy\right. \qquad (3.28)$$
$$\left. + \int_{\frac{1}{2}}^1 y. \frac{(2.1-2y)}{(1-0)(1-1/2)}\, dy\right]$$

$$\Delta_j = \left\{\int_a^c x \,. \left[\frac{(x-a)^2}{(b-a)(c-a)}\right] . \left[\frac{2(x-a)}{(b-a)(c-a)}\right] dx\right.$$
$$\left. + \int_c^b x \,. \left[1 - \frac{(b-x)^2}{(b-a)(b-c)}\right] . \left[\frac{2(b-x)}{(b-a)(b-c)}\right] dx \right\}$$
$$- \left\{\frac{a^2 - 2c^2 + ac}{3(a-b)}\right.$$
$$\left. + \frac{[-(b^2 - 2c^2 + bc)]}{3(a-b)}\right\} \left[\int_0^{\frac{1}{2}} y. \frac{(2y-2.0)}{(1-0)(1/2-0)}\, dy\right.$$
$$\left. + \int_{\frac{1}{2}}^1 y. \frac{(2.1-2y)}{(1-0)(1-1/2)}\, dy\right]$$

$$\Delta_j = \frac{2a^2 + 2b^2 + c^2 - 3ab - ac - bc}{15(b-a)}$$

The statement referred has reasoning on research on the Institute Scientific Information (ISI) and in Scopus Database, that reveals no results of permutations' applications for linear transformations to generate triangular distributions correlated by Gini parameters. The formulation for permutations or interchange of distributions in linear transformations is in Equation (3.29).

The variable y in Equation (3.29) represents the triangular $T_y(a_y, b_y, c_y)$ that we want to simulate from a distribution of a variable x, which also follows a triangular distribution $T_x(a_x, b_x, c_x)$. As seen in Equation (3.29), the method technology has a simple structuring, where the indexes 0 and 1 in the equation are the beginning and end of the two triangular distributions in strategic regions.

$$y\ = y_0 + (y_1 - y_0).(x_1 - x_0)^{-1}.(x - x_0) \quad (3.29)$$

The triangular distributions must have a segmentation logic to the transformation. The segmentation in strategic regions means finding equivalent areas between the triangular distributions to transformations. This equivalence must occur, both in the accumulated densities and in the ancestry or descent. For example, in Figure 6, the hatch region of the distribution x is equal to the hatch region of the distribution y, where both areas ascent in their respective triangles and restrict 50% of their distributions.

In the case of the example, after verifying the accumulated percentage of the hatched region of the triangular distribution of x in its ascending form (the area 1 of the triangle of x), then we must calculate the respective numerical limits x_0 and x_1. Subsequently, according to the cumulative percentage and variation of the region, which must both be equivalent, then the numerical limits y_0 and y_1 must be calculated. Finally, we must apply Equation (3.29) for converting the crosshatch region of x into the crosshatch of y.

In the continuation of the segmentation of the example of Figure 6, it is evident that the segments with descending triangles and rectangles are equivalent. Therefore, they determine the permutation. The complementation to end the permutation in the region marked with two can interfere in the correlation obtained because the triangles differ in the ascent and descent. Therefore, the transformation must occur randomly or still undergo other internal rectangular segmentations in these regions before randomizing.

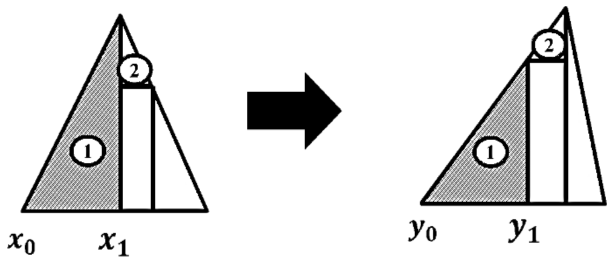

Figure 6 – Segmented permutation in a linear transformation
Source: authors

The example presented is for generations of direct correlations. Thus, we can intuitively conclude that inverse correlations can be obtained by inverting one of the pairs. An expressive advantage of this method is that it can be easily programmed (for example, in Scilab®, Python®, and others).

In tests of this method, it was possible to verify a significant observation: the height of the density function is ever equal to 2 / $(b - a)$, and according to the triangular values of the application of opening of the triangles of approximately 50,000 units, we have very high Gini correlation limits obtained of at least 99.9%. If we did not desire a high correlation, we must randomize the data in the desired proportion.

3.6 Applications in Photovoltaic Projects and Small Businesses

The bias of the adjacent methods for applications in selecting projects is highly strategic because it allows a more plausible justification for research from a social perspective, especially with a target for applications adjacent to micro and small companies. I.e., there are strategic calls for use by a sector without political and legislative. This sector is experiencing an eternal economic and financial recession, caused by the absence mentioned of medium-and-long-term policies and legislation (CARVALHO et al., 2018; MALAQUIAS and HWANG, 2016).

Another segment that considerably increases the justifications for research is applications in sustainable, renewable and, environmentally friendly projects. Therefore, research problems with their justifications will become much more relevant socially, but only if the structures of the adjacent methods contemplate the change mentioned. Also, the research's justifications and problems have mainly based on the impossibility of using variance as a risk metric for a project portfolio. Another aspect at the centre of this discussion is the advantages of using Gini and Gini-CAPM in specific scenarios (DAVIS and MARTIN, 2014; GAWEL et al., 2017; SHEZAN et al., 2016; SOLANGI et al.., 2011; WESSEH and LIN, 2016).

Several other justifications based on the need for solutions to issues about the eminent need for sustainable projects in general are: the much sought after reduction of carbon emissions in the atmosphere; the help to correct the current energy matrix always aiming at more significant uses of renewable and ecologically correct energy; the minimizing conflicts over fossil energy and all its consequences; and others (BECKER, and FISCHER, 2013; HUENTELER, 2014; JACOBS et al.., 2013; JENNER, GROBA, and INDVIK, 2013; QUEIROZ, 2016; WONG, BHATTACHARYA, and FULLER, 2010).

From the economic and financial point of view, we have: the eminent need to increase the competitiveness of micro and small companies worldwide, but mainly of developing countries and their second and third sectors; the eternal recession of the economy of developing countries, such as Brazil, mainly of their micro and small companies (due to the lack of a correct policy and legislation in the medium and long term); the extensive range of families in extreme poverty in Brazil and the world, as this occurs a lot due to the inefficiency of the competitiveness of the micro and small companies in question; and others (CARVALHO et al., 2018; MALAQUIAS, and HWANG, 2016).

Concerning the technical point of view and, more objective for the program that houses this research, the justifications continue to increase because research based on the Institute Scientific Information (ISI) and Scopus database reveal structural gaps. I.e., there are no structured methods, models, procedures, or programming for applications adjacent and according to the scope in question. Still, the thesis and publications in scientific journals that this research will generate and other potential outputs will likely be much more plausible in the technical sense.

4 RESULTS AND DISCUSSIONS

In this Section, we present the results and discussions regarding the practical applications of the methods developed by this research and all its technology. In fact, these developments we grouped into two methods: the first is a method to project portfolio selection using criteria Gini-CAPM and Gini-semi-variations, which also is analytic to computational cost reduce (Method I); and the second is a method to project portfolio selection similar to the previous one, but to solar energy by photovoltaic cells, and able to consider interdependence in ROI (Method II). Respectively, the applications of these methods are in Section 4.1 and Section 4.2.

4.1 Applications and Discussions about the Method I

The risk-free rate R_f adopted for the study is a monthly approximation value of the SELIC (Special Settlement and Custody System) for the last months (0,50 %). SELIC is the main parameter to define returns on government bonds in Brazil. Also, in modelling, we consider that when the company starts borrowing, R_f increases, by-product with $3^{(\sum_{j=1}^{n} x_j - 1)}$, to try to reflect the reality of the rate increase with the increase of a micro company's debt (in fact, R_f = R_f until $\sum_{j=1}^{n} x_j \leq 1$, or to the resultant of the product by $3^{(\sum_{j=1}^{n} x_j - 1)}$ to $\sum_{j=1}^{n} x_j > 1$).

The other main inputs for the method are in Table 1, where in the first column is projects identifications. In the second column is the ROI of each project, which we calculate based on triangular distributions parameters predicted by the micro-enterprise. The triangular distribution parameters are in the third, fourth and fifth columns and correspond to the pessimists, likeliest, and optimistic values. The sixth column of the table presents the amounts of initial investments required for each project.

However, the company does not have the capital to invest in all projects, but only R$ 120,000.00. This maximum value of investment capital together with the individual values give rise to the relative maximum participation vector z_j (in the seventh column), which determines the level of risk related to the participation of the projects. That is, as the value needed for each investment in a project rises or the total capital available falls, the relative risk will increase. Moreover, the reverse logic also applies.

In the eighth column of Table 1 are the Gini risk values that we calculated using the parametric algebraic formulation presented in Section 3.5.1 And other important information related to R_f is that it is a source of investment to the capital slack and for applications of the cash flows of the projects. The last condition does not change the selection results, as it raises all the ROI values precisely in the dimension R_f. And these two lasts conditions were implemented to simulate a situation closer to the reality in professional financial management.

Projects	ROI	Triangular Distributions			Investments R$	Participation	Gini Risk
		Pessimistic	Likely	Optimistic			
A	1.31%	-5.20%	1.26%	7.88%	29,855.6	24.88%	1.53%
B	1.33%	-8.64%	1.35%	11.27%	29,933.9	24.94%	2.32%
C	1.31%	-5.66%	1.66%	7.94%	29,855.6	24.88%	1.59%
D	1.33%	-8.71%	0.80%	11.89%	29,933.9	24.94%	2.40%
E	1.31%	-5.36%	1.26%	8.04%	29,840.5	24.87%	1.56%
F	1.33%	-8.63%	1.25%	11.36%	29,926.3	24.94%	2.33%
G	1.31%	-5.80%	2.26%	7.48%	29,798.1	24.83%	1.56%

Table 1 – Main inputs to the Method I

With the definitions and performed calculations, we carried out the first three steps according to the pseudocode of Figure 3. Thus, the next step in method application is to design a dynamic matrix named “Gini-variation.” In sequence, we must obtain the optimal global portfolio to $0 \leq x_j \leq 1 \; and \; 0 \leq x_j z_j \leq z_j$. “Sharpe's Gini-CAPM portfolio is the index that best represents the market, and is the global optimum” (HODOSHIMA and OTSUKI, 2019).

In our project's scenario, we assume that the Sharpe ratio portfolio is a reasonable representation of the market, and therefore it provides a good benchmark. On other hand, Table 2 shows the results of continuous global optimal x_j^* in the second column. The third column presents the real participation corresponding to the optimal global for all the projects. These last values are calculated simply by multiplying the decision variables x_j^* with the upper bound vector z_j. Finally, in the last three lines, the table presents the main information related to the optimal portfolio: the ROI, the Gini risk, and the Sharpe-Gini CAPM.

In fact, from the results of the real participation $x_j^* z_j$, we can see excess in the sum concerning 100%. This indicates that if the optimal global portfolio were feasible, it would only be possible with financial leverage. Also, it is essential to remember that the portfolios must receive complementation by the product of the risk-free rate R_f by investments slack to obtain Table 2 values. Nevertheless, this is only when the sum of the portfolio relative shares is less than 100%, and from there R_f grows exponentially by multiplied by $3^{(1-\sum_{j=1}^{n} x_j)}$.

Projects	Optimal Global Participation x_j^*	Real Global Optimal $x_j^* z_j$
A	0.761	18.925%
B	0.333	8.304%
C	0.703	17.493%
D	0.310	7.740%
E	0.725	18.023%
F	0.330	8.220%
G	0.730	18.118%
	ROI	1.291%
	Gini Risk	0.655%
	Sharpe-Gini CAPM	1.209

Table 2 – Continuous global optimum in Method I application

In Figure 7 are the main results found until now. It is possible to observe in the graphic the continuous efficient frontier of the system, where is the optimal global portfolio x_j^*. The graph also shows the line similar to of capital market and the feasible region of project portfolios. Assuming that the investment slack between the maximum available and the applied capital, we will invest to R_f, then we can observe a typical deleveraging behaviour in the first quadrant to the left of the efficient frontier.

In this first quadrant to the left of the efficient frontier, it exists practically a continuous linear frontier up to the point where the number of projects consumes all the resources of the micro company itself available for investment. However, the boundary changes direction from that point, indicating greater proportionate risk and with a typical concavity of the Markowitz diversification benefit. Moreover, in the latter case, the boundary convex encounters a supposed discrete frontier at the point where are all the projects in a portfolio.

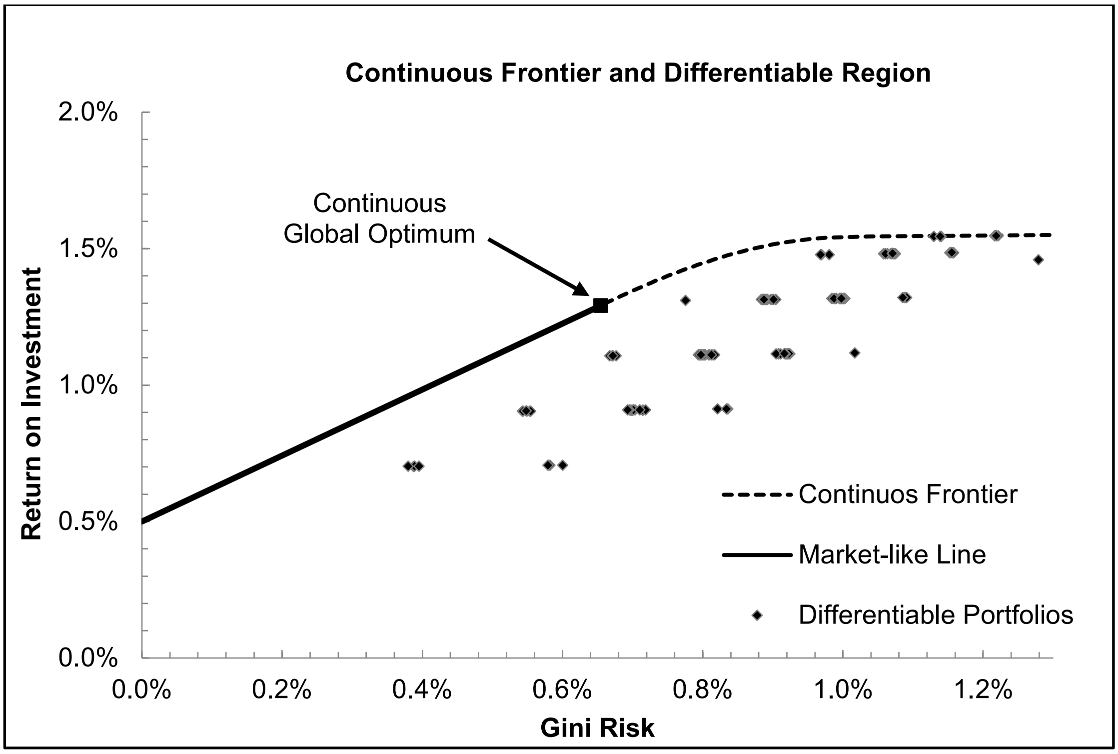

Figure 7 – Continuous frontier and differentiable region to Method I

Source: authors

The differentiable and discrete region $\left(\text{to } x_j \in [0\ 1] \text{ and } 0 \leq x_j z_j \leq z_j\right)$ present in Figure 7 has a set with 127 options, and it is the focus of this study. The reason is that although the continuous boundary dominates the differentiable region, it does not attend to the assumption of real possibilities in the scenario. The line similar to the capital market, on the other hand, has a very typical behaviour, that is, it starts in R_f and ends in $x_j^* z_j$.

At that moment, we overcame the first eight steps of the proposed method in Figure 3 pseudocode. Furthermore, with the parameters and variables found in the method executions so far, it is possible to calculate the values of the selection criteria. The first is GS_s, and must be obtained for all differentiable portfolios of $s = 1{:}p$. According to the CAPM, using the Sharpe index, we can select portfolios with a favourable probability of returns per risk unit, a lower probability of causing losses, among others. The set with Equation (3.5), restring by (3.6) and Equation (3.7), presents the formulations for the calculation of GS_s. At this moment, we must calculate the non-diversifiable Beta-Gini risk values GB_s, where we assumed the strategy for avoiding losses (minimizing GB_s). The formulation of variable GB_s is given by Equation (3.8), also restrained restring by (3.6) and Equation (3.7).

The third among the main variables of the method is μ_s^3, the third moment. As skewness indicates the direction and relative magnitude of the risk of distributions, it can indicate opportunities. In probability distribution functions, players generally prefer positive skewness to take advantage of more significant deviations. However, it is wiser to have a lower probability of loss, which is the strategy adopted in applying the method. Equation (3.10) shows the formulations for μ_s^3.

Finally, the fourth variable used as a decision criterion in the method is SD_s, the second-order stochastic dominance. A stochastic dominance of a portfolio j over another j' indicates that the first always have greater probabilities of satisfactory returns. Alternatively, the stochastic dominance means that portfolio j is less likely to generate losses. To define stochastic dominance, we use the conditions Equation (3.10) and Equation (3.11), and then we account for the number of portfolios dominated by each of the few dominants.

After calculating SD_s we perform the twenty steps of the method according to Section 3.3. At this moment, the definition of the system's discrete and efficient boundary is strategic, that is, to $x_j \in [0\ 1]$. This boundary synthesizes all the selection criteria used in the method, and it includes some of the portfolios with the highest GS_s indices and some with the lowest non-diversifiable risks GB_s. Furthermore, the definition of the efficient boundary can adequately approximate stochastic dominance SD_s and represent the portfolios with the best statistical probabilities of μ_s^3. Figure 8 presents the boundary that summarizes the four selection criteria used.

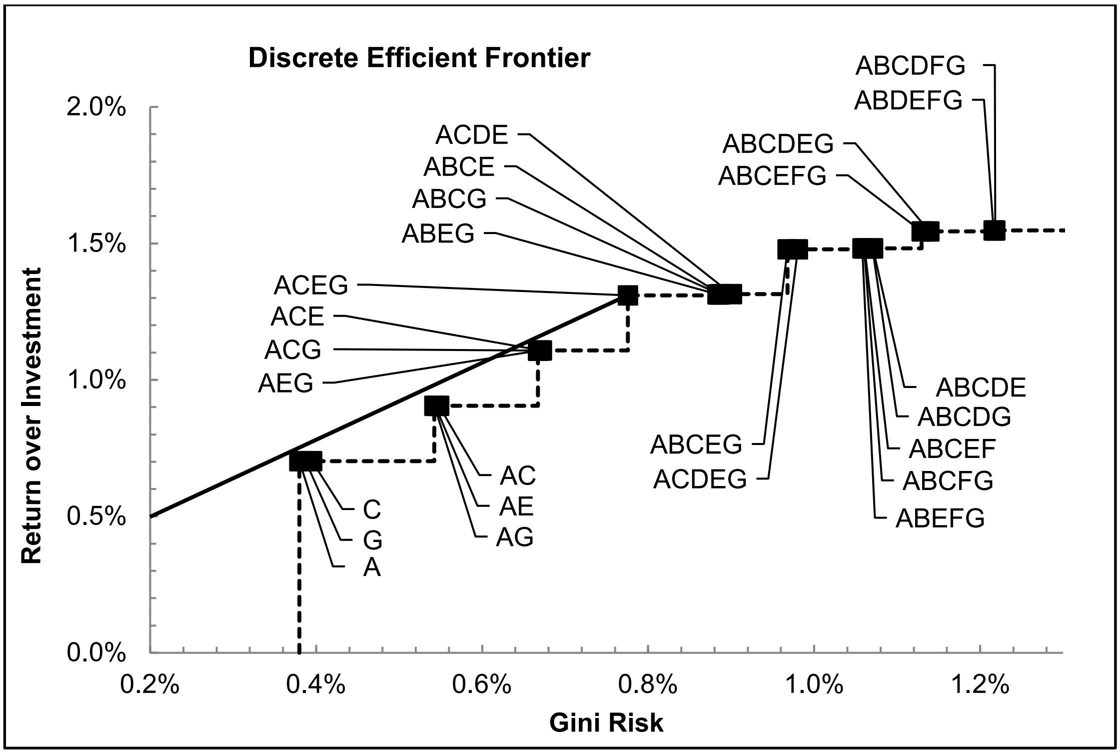

Figure 8 – Discrete and efficient frontier to Method I
Source: authors

Finally, Table 3 presents the results for GS_s, GB_s, μ_s^3 and SD_s, where are only the 40 best portfolios that meet the restriction of the sum of initial investment $\leq$ R\$ 120,000.00. In the first table column are the descriptions of the best portfolios according to the new method. The following four columns present the values found, respectively, for the variables Sharpe-Gini GS_s, Beta-Gini GB_s, Skewness-Gini μ_s^3 and Stochastic-Dominance-Gini SD_s. The sixth column, which determines the descending order of presentation, shows the results of the AHP multicriteria tool.

Best Portfolios	Sharpe-Gini GS_s	Beta-Gini GB_s	Skewness-Gini μ_s^3	Stochastic-Dominance-Gini SD_s	AHP Results
ACEG	1.0441	0.7465	-0.3792	63.0000	0.0102
AEG	0.9097	0.5665	-0.3117	29.0000	0.0090
CEG	0.8978	0.5518	-0.4549	28.0000	0.0090
ACG	0.9051	0.5612	-0.4581	0.0000	0.0084
ABCG	0.9179	0.6480	-0.3534	0.0000	0.0084
BCEG	0.9135	0.6385	-0.3520	0.0000	0.0083
ACFG	0.9159	0.6470	-0.3247	0.0000	0.0083
CEFG	0.9116	0.6376	-0.3235	0.0000	0.0083
ABEG	0.9204	0.6533	-0.2420	0.0000	0.0083
AEFG	0.9184	0.6523	-0.2135	0.0000	0.0083
ACE	0.9043	0.5601	-0.0880	0.0000	0.0082
ACDG	0.9039	0.6421	-0.1942	0.0000	0.0082
CDEG	0.8997	0.6326	-0.1935	0.0000	0.0082
ABCE	0.9174	0.6469	-0.0732	0.0000	0.0082
ACEF	0.9154	0.6460	-0.0451	0.0000	0.0082
ADEG	0.9063	0.6474	-0.0840	0.0000	0.0081
ACDE	0.9035	0.6410	0.0816	0.0000	0.0080
AG	0.7462	0.3812	-0.4082	9.0000	0.0080
BCFG	0.8253	0.5390	-0.3108	0.0000	0.0080
ABFG	0.8304	0.5538	-0.2116	0.0000	0.0079
BEFG	0.8271	0.5443	-0.2110	0.0000	0.0079
EG	0.7370	0.3717	-0.4037	7.0000	0.0079
BCDG	0.8166	0.5340	-0.1940	0.0000	0.0079
CG	0.7316	0.3664	-0.5801	0.0000	0.0079
CDFG	0.8152	0.5331	-0.1689	0.0000	0.0078
ABCF	0.8283	0.5474	-0.0600	0.0000	0.0078
BCEF	0.8251	0.5380	-0.0600	0.0000	0.0078
ABDG	0.8215	0.5488	-0.0951	0.0000	0.0078
BCG	0.7628	0.4532	-0.4080	0.0000	0.0078
BDEG	0.8184	0.5393	-0.0949	0.0000	0.0078
ADFG	0.8201	0.5479	-0.0700	0.0000	0.0078
DEFG	0.8169	0.5384	-0.0699	0.0000	0.0078
CFG	0.7608	0.4523	-0.3760	0.0000	0.0078
ABEF	0.8301	0.5527	0.0407	0.0000	0.0078
ABG	0.7700	0.4679	-0.2863	0.0000	0.0078
BEG	0.7654	0.4585	-0.2850	0.0000	0.0077
AFG	0.7679	0.4670	-0.2544	0.0000	0.0077
ABCD	0.8195	0.5424	0.0544	0.0000	0.0077
EFG	0.7634	0.4576	-0.2532	0.0000	0.0077
BCDE	0.8164	0.5330	0.0541	0.0000	0.0077

Table 3 – Final results for selection by Method I

To obtain the AHP results, the micro company that participated in the study determined that GS_s should have three times more weight than other criteria, and the other criteria should be of equal importance. In data processing, before applying AHP, the GB_s values were multiplied by the constant (-) 1 (to invert the values, and the logic of bigger is better to be predominate). In sequence, in the results, we plus to the constant 2 to move it away from zero (to avoid divisions involving minimal values). Finally, we treated variable μ_s^3 identically to GB_s, and variable SD_s we linearized between constants 1 and 2 so that its results would have a similar impact to the others. The results of GS_s went to AHP without modification.

4.1.1 Preliminary Results Validations of the Method I

According to Table 3, the best two options are ACEG and after AEG by the new method. The method neither presupposes nor disposes of historical data; thus, one way to validate the results at a preliminary level is using simulations. For this, we generated 10^6 data for each project in the simulations according to their probability distributions. We then compare the two portfolios selected by the new method with others selected by Net Present Value (NPV) and Internal Rate of Return (IRR). The reason for the choice of NPV and IRR is that "these models are the most used contemporaneously for the selection of asset portfolios, including discrete investment projects (LIU and REYNOLDS, 2017). In this preliminary validation of the method, we do not run comparisons with methods using the variance metric because the central premise of this research is that we should not use variance to analyze non-normal probability distributions.

Respectively, we selected 15 project portfolios using the NVP model and 15 using the IRR, and we also used an exclusive selection not to generate portfolio redundancies. The selection was according to the highest level of use of each of the two options, i.e., first by NVP and then by IRR. Furthermore, as established, we applied an initial investment limit for the competitors, as well as for the proposed new method of R$ 120,000.00. The restriction of initial investment imposed that the portfolios had a maximum of 4 components.

In the simulation, we calculate the probabilities of the optimal portfolios concerning obtaining: of returns below – R_f, of returns below 0, and returns above + R_f. The first two results seek to identify portfolios with the lowest probabilities of losses. The third result seeks to identify portfolios with greater probabilities of excess returns. Lastly, we made an overall rank in the evaluation, which is the arithmetic means of the ranks of the three results.

Table 4 presents in its first column the portfolios' compositions in ascending alphabetical order. The second column presents the method identification that selected the portfolios, where MT is the method proposed by this research. The third and fourth columns show fundamental criteria, wherein the two, there is the probability of the portfolios analysed according to the excellence in minimum chances of ROI for levels below -R_f and below 0. In The third and fourth columns, the portfolios selected by the new method occupy the first and second place in the ranking. For return probabilities smaller than – R_f the first ranked is AEG and the second is ACEG (with 8.443% and 9.121%). For probabilities of returns less than 0, the first ranked is ACEG and the second is AEG (16.729% and 17.162%).

The fifth column of Table 4 presents the results according to the probabilities of satisfactory levels of ROI. Again, the company applying the method defined the criteria, where the ROI probabilities analysed in this criterion were the highest that R_f. In this case, the portfolios selected by the proposed method also have good ratings. The classification of ACEG got first place (with 72.460%), and the classification of AEG received sixth place (with 69.837%). The sixth column of Table 4 presents a final rank for each portfolio according to the average rating of all criteria analysed. Therefore, this last column shows the final result synthesized, where ACEG and AEG portfolios occupied first and third place in the study, which corroborates for statements about the excellence of portfolio selection by the proposed method.

Portfolio	Method of Selection	Probability $\leq -R_f$	Probability ≤ 0	Probability $\geq R_f$	General Rank
ABCD	NVP	14.891%	22.520%	68.023%	21
ABCE	MAR	12.127%	19.864%	69.997%	4
ABCF	NVP	14.630%	22.276%	68.203%	13
ABCG	MAR	12.117%	19.855%	70.003%	2
ABDE	NVP	14.835%	22.469%	68.059%	18
ABDF	NVP	16.879%	24.332%	66.726%	29
ABDG	NVP	14.827%	22.463%	68.062%	17
ABEF	NVP	14.572%	22.223%	68.240%	12
ABFG	MAR	14.565%	22.217%	68.243%	11
ACDE	MAR	12.490%	20.226%	69.719%	9
ACDF	NVP	14.930%	22.558%	67.994%	23
ACDG	MAR	12.480%	20.217%	69.724%	8
ACEF	MAR	12.176%	19.913%	69.958%	6
ACEG	MT	9.121%	16.729%	72.460%	1
ACFG	MAR	12.166%	19.904%	69.963%	5
ADEF	NVP	14.874%	22.507%	68.030%	20
ADEG	MAR	12.412%	20.151%	69.773%	7
ADFG	MAR	14.867%	22.501%	68.032%	19
AEG	MT	8.443%	17.162%	69.837%	3
BCDE	NVP	14.978%	22.604%	67.959%	26
BCDF	NVP	17.000%	24.441%	66.648%	32
BCDG	NVP	14.971%	22.598%	67.962%	25
BCEF	NVP	14.719%	22.361%	68.138%	16
BCFG	MAR	14.711%	22.355%	68.141%	15
BDEF	NVP	16.954%	24.400%	66.675%	31
BDEG	MAR	14.915%	22.547%	67.998%	22
BDFG	NVP	16.948%	24.396%	66.677%	30
BEFG	MAR	14.654%	22.302%	68.177%	14
CDEF	NVP	15.018%	22.641%	67.931%	28
CDEG	MAR	12.585%	20.322%	69.641%	10
CDFG	MAR	15.010%	22.635%	67.933%	27
DEFG	MAR	14.954%	22.584%	67.969%	24

Table 4 – Monte-Carlo simulations results to Method I

Therefore, according to analyses based mainly on the numerical results of simulations, the new method will select significantly favourable portfolios compared to traditional and more used methods contemporarily. Among the beneficial characteristics to be expected for portfolios selected by the new method, the following stand out the lowest loss probabilities and the highest probabilities of ROI favourable.

4.2 Applications and Discussions about the Method II

This Section evaluates the specificities scenario where the method is applied, executes the application in a real case, and uses the application for results discussions. The scenario assessment is necessary to identify the SEPC projects' characteristics to adjustments in the method, according to the dimensions, the inputs features, the country legislation, the strategies adopted, among others. The results and discussions present inferential graphs and tables, always accompanied by interpretations to support selection.

4.2.1 Scenario Specificities

According to the scope of the research, to apply the method, we chose projects options for small and micro companies, where a monthly consumption level is approximately 3,500 kWh. This level of consumption is strategic to comply with the range that receives the most significant incentives from the country's legislation. The enterprise that accepted participating in the study is a small electrical installations company located in Machado city, Minas Gerais, Brazil. It saw the opportunity to offer finance to small and micro businesses in the region in installations of SEPC, which has provided some service in recent years. The financing proposals offer legal procedures for authorization, the equipment, installations, and maintenance during the amortization period.

The repaying period to financing is 144 months, with payments at the end of all of them and in amounts proportional to only 85% of the price charged by the energy concessionaire. Therefore, at the financing end to micro and small companies, the installation will be fully amortized and still have a useful life of at least 13 years (according to guarantees from the equipment suppliers). Furthermore, the micro and small businesses will have the opportunity to use the equipment depreciation and the reduction in interest on the financing to leverage their real profits.

However, the benefits are not only concentrated for the environment and for the micro and small companies that will be able to install the SEPC, but also for the small electrical installation company that is offering the financing. It will also significantly benefit from returns on investment with a high degree of attractiveness and in a low-risk activity. The ROI values calculated by the small enterprise and by the method are in Section 4.2.2.

We have not discarded in the method the form of the small financing company's ROI predictions. However, we consider that form has excellence, and its values are compared with method pricing to identify opportunities. Also, we used both ROI values to compose a first cut-off line for the study, but in conjunction with high levels of Gini risk, very unfavourable Gini correction, necessary initial investments far above average, and production capacity. In this case, among 19 initial projects options, only 11 remained eligible.

We used the Maximin strategy to project a less unfavourable result among the options in the worst scenario projection of cash flows. Thus, if the worst possibilities for the investment materialize, the results will still be satisfactory in the investment. Alternatively, the tendency for the results to be better than projected has a significant probability. Within this strategy, we projected that the price of kWh will not undergo readjustment, that the growth trend will be minimum, and that the increase in initial investment will occur in line with the dollar's rise.

Other relevant pieces of information for the projection of cash flows to calculate the ROI amounts is the deduction of all IRPJ, ICMS, CSLL, COFINS, PIS (Corporate Income Tax, Tax on Circulation of property and Provision of Services, Social Contribution on Net Income, Contribution to Social Security Financing, and Social Integration Program), and a monthly maintenance fee of 0.5%. We defined the lower limit of the adjacent triangular distributions parameters to calculate the ROI values when the worst possibilities were all gathered. However, when only the most probable possibilities (among them some pessimists), the result was assumed as the most possible value for the distribution. Finally, when we eliminated all pessimistic predictions, the adjacent result was considered the upper limit.

For projections, we used the ARIMA method (Autoregressive Integrated Moving Averages). The setting configuration of non-seasonal was of p = 0, d = 0, q = 1, and of the seasonality of P = 0, D = 1, Q = 1. The setting (0 1 1) (0 1 1) would have a superior fit but would not minimize the trend of consumption growth in our Maxmin strategy. However, other simpler prediction models are perfectly acceptable (for example, the Classic Decomposition Additive or Multiplicative). The reason is that unlike observed for the equity markets, the forecast consumption of energy is relatively trivial. Also, simpler models will be more intuitive and do not require specialist software.

4.2.2 Results and Discussions about Method II

A significant contribution of the Gini-CAPM is to minimize the possibility of prediction mistakes based on history. For example, when we choose a historical number of periods that do not reflect reality well, the regressive pricing of the methodology adjusts the error. Mainly based on the tangent, or the first-order derivative, the risk we called Beta-Gini. Likewise, if we used methods in theoretical evaluations, the results would be identical between distributions and pricing parameters.

Furthermore, unlike most situations in project evaluations, in the applications of the new method, there is data history of its main input, which is the energy consumption of each of the micro and small companies studied. Table 5 presents the business sectors of micro and small companies and determines an alphanumeric encoding to each. The table also shows the results of average ROI predictions according to the know-how of the small company offering the financing. It is possible to observe that, given the scope of specifications for potential project searches, the funder ended up selecting more than one option within specific sectors. The strategy can be harmful because it is a potential obstacle to diversification.

Identification	Microenterprise Sector of Activity	ROI Annual Mean Predicted by Financier
A	Construction company	17.14%
B	Coffee production farm	23.09%
C	Coffee production farm	27.31%
D	Food industry for export	27.92%
E	Food industry for domestic market	22.79%
F	Retail supermarket	21.02%
G	Retail supermarket	23.55%
H	Wholesale and retail supermarket	19.52%
I	Mall of medium-sized	18.49%
J	Metal mechanic industry	22.20%
K	Beef slaughter industry	28.13%

Table 5 – Identifications and annual ROI estimated of Method II projects

Table 6 presents the main inputs for the method. It contains the results of ROI projected by method, the parameters of the obtained triangular distributions, the necessary initial investments, and the vector z_j of maximum possible participations. It is also necessary to add that the risk-free rate $-R_f$ adopted for the study is an approximate average value of the SELIC (Special Settlement and Custody System) for the last thirty months and annualized (3.00% per year). SELIC is used as the main parameter to define returns on government bonds in Brazil.

Projects	ROI	Parameters of Triangular Distributions			Investments	Participation
		Pessimistic	Likely	Optimistic		
A	20.39%	8.73%	19.84%	32.61%	129,470	8.69%
B	21.32%	9.49%	21.69%	32.77%	133,983	9.00%
C	24.41%	16.01%	24.41%	32.81%	149,475	10.04%
D	25.00%	16.77%	25.10%	33.13%	137,558	9.24%
E	20.46%	9.72%	20.79%	30.87%	123,472	8.29%
F	23.06%	14.07%	23.02%	32.09%	133,397	8.96%
G	25.35%	13.78%	25.55%	36.72%	143,165	9.61%
H	21.95%	9.59%	22.02%	34.24%	131,635	8.84%
I	21.29%	8.61%	20.49%	34.77%	132,802	8.92%
J	20.08%	9.14%	20.20%	30.91%	131,144	8.81%
K	25.54%	17.01%	26.13%	33.49%	143,091	9.61%

Table 6 – Main inputs for Method II

Other important information related to R_f is that it is a source of investment, both to the investment capital slack (when at least one of the projects is not in the selected portfolio) and for applications of the cash flows of the projects themselves. The last condition mentioned does not change the selection results, as it raises all the ROI values precisely in the dimension R_f, and was implemented to simulate a situation closer to reality in professional financial management. The suppositions also imply that the investor has initial capital available to implement all the projects. It is worth noting that a minor total capital will mean more significant risks.

With the definitions, predictions, and calculations so far, we carried out the first three steps according to the pseudocode of Figure 4. The next step is to calculate the Gini correlations between the energy consumption values of the micro or small companies studied. Table 7 shows these values, where it is possible to observe that, unlike the Pearson correlation, the Gini between jj' and $j'j$ can be different.

	A	B	C	D	E	F	G	H	I	J	K
A	1.00	-0.09	-0.11	-0.17	0.16	0.09	0.09	0.09	0.07	0.44	-0.22
B	-0.09	1.00	0.44	-0.06	-0.24	-0.35	-0.33	-0.33	-0.11	-0.28	-0.05
C	-0.10	0.43	1.00	-0.07	-0.22	-0.30	-0.34	-0.32	-0.09	-0.25	-0.07
D	-0.17	-0.05	-0.08	1.00	-0.36	-0.13	-0.15	-0.08	-0.22	-0.38	0.43
E	0.15	-0.24	-0.22	-0.35	1.00	0.17	0.17	0.08	0.21	0.35	-0.35
F	0.09	-0.36	-0.31	-0.15	0.17	1.00	0.44	0.42	0.34	0.17	-0.16
G	0.10	-0.32	-0.33	-0.15	0.17	0.44	1.00	0.45	0.35	0.18	-0.12
H	0.09	-0.33	-0.32	-0.08	0.08	0.43	0.44	1.00	0.37	0.18	-0.08
I	0.07	-0.10	-0.08	-0.21	0.22	0.34	0.36	0.38	1.00	0.14	-0.25
J	0.43	-0.28	-0.26	-0.38	0.36	0.18	0.18	0.17	0.13	1.00	-0.33
K	-0.22	-0.06	-0.08	0.44	-0.35	-0.16	-0.13	-0.07	-0.24	-0.34	1.00

Table 7 – Gini correlation between projects in Method II application

Table 7 also allows abstracting essential information concerning observed levels of correlation. As expected, projects related to micro-enterprises in the same sector have high positive and undesirable values, making diversification difficult (example: between F, G, and H). On the other hand, a first signal that selection may offer significant benefits were some negative correlations found (example: between D and J), which may help to minimize total risks obtain good diversification between others.

Now in method application, it is possible to design a dynamic matrix (similar to the variance and covariance), in which the matrix name "Gini-variation and Gini-covariation $\Delta_{jj\prime}$" can be given. Thus, we completed the first 11 steps of the Figure 4 pseudocode of the new method. Subsequently, it is possible to calculate various strategic parameters. Initially, we must obtain the optimal global portfolios similar to the Sharpe-Gini-CAPM, to continuous $(to\ 0 \leq x_j \leq 1\ and\ 0 \leq x_j z_j \leq z_j)$, and to discrete $(to\ x_j \in [0\ 1]\ and\ 0 \leq x_j z_j \leq z_j)$.

Table 8 shows these results, where the second column presents the upper limits investment z_j. In the third and fourth are, respectively, the continuous and discrete optimal. Table 8 shows the variables participations corresponding to the global and discrete optimal portfolio in the fifth and sixth columns. These values are calculated simply by multiplying the decision variables with the upper bound vector z_j. Finally, in the last three lines, the table presents the main information related to the two optimal portfolios: the ROI, the Gini risk, and the Sharpe-Gini CAPM.

It is noteworthy that, if numerically the results of the three variables do not seem very distant (ROI, Gini risk, and Sharpe-Gini between Global Optimal and Discreet Optimal), for returns and financial risks of high investments they are significantly high. It is also important to remember that the portfolios must receive complementation by the product of the risk-free rate R_f by investments slack to obtain Table 8 values. At that moment, in the new method pseudocode, we performed the first 17 steps in the application.

Projects	Upper Limit z_j	Optimal Global Participation x_j^*	Optimal Discreet Participation $x_j \in [0\ 1]$	Real Global Optimal $x_j^* z_j$	Real Discreet Optimal $x_j z_j$
A	8.69%	11.75%	0.00%	1.02%	0.00%
B	9.00%	53.06%	100.00%	4.77%	9.00%
C	10.04%	84.81%	100.00%	8.51%	10.04%
D	9.24%	88.09%	100.00%	8.14%	9.24%
E	8.29%	64.67%	100.00%	5.36%	8.29%
F	8.96%	51.12%	100.00%	4.58%	8.96%
G	9.61%	29.43%	0.00%	2.83%	0.00%
H	8.84%	17.93%	0.00%	1.59%	0.00%
I	8.92%	3.25%	0.00%	0.29%	0.00%
J	8.81%	57.62%	100.00%	5.07%	8.81%
K	9.61%	80.22%	100.00%	7.71%	9.61%
			ROI	13.11%	15.74%
			Gini Risk	0.25%	0.35%
			Sharpe-Gini CAPM	41.078	36.637

Table 8 – Found optimal global to Method II

A satisfactory understanding of the method can occur using graphic resources. Figure 9 presents some of the main results obtained so far between the Gini risk x ROI dimensions. It is possible to observe in Figure 9 the continuous efficient frontier of the system, where the optimal global portfolio is x_j^*.

According to the assumption that the investment slack between the maximum necessary and the effective capital will be applied in R_f, then a typical deleveraging behaviour we can observe in the continuous efficient frontier, similar to compositions with the global optimum x_j^*. In case the continuous frontier starts at R_f, runs linearly up to x_j^*, and from there, it begins to configure itself with a typical concavity (since there is no leverage assumption at R_f), until it touches the discrete differentiable region. The continuous frontier of the system is, in fact, utopian and presented because it contains the optimal market portfolio x_j^*, and to help didactically understand the new method.

It is significantly essential and present in Figure 9, is the differentiable and discrete region $to\ x_j \in [0\ 1]\ and\ 0 \leq x_j z_j \leq z_j$, composed of 2047 options. This is because, although the continuous boundary domine the differentiable region completely (as it should be), it fulfils the factual assumption of actual possibilities in the scenario. Therefore, it is here that the analyses in the method and the selection itself are concentrated. Furthermore, the optimal portfolio chosen in applying the method (BCDEFJK) is also in the differentiable region.

With the parameters and variables defined, esteemed, and calculated in the method executions so far, it is possible to calculate the values of the selection criteria. The first is GS_s, and must be obtained for all differentiable portfolios of $s = 1{:}\,p$. With the index, according to the CAPM and Gini-CAPM theories, we can select portfolios with the best probabilities of excess of satisfactory returns per risk unit, lower probabilities of cause losses, between others. In Section 3.4, the set with Equation (3.20) and restrictions (3.21) and (3.22) presents the formulations for the calculation of GS_s. In the pseudocode of Figure 4, calculation corresponds to steps 16, 17 and 18.

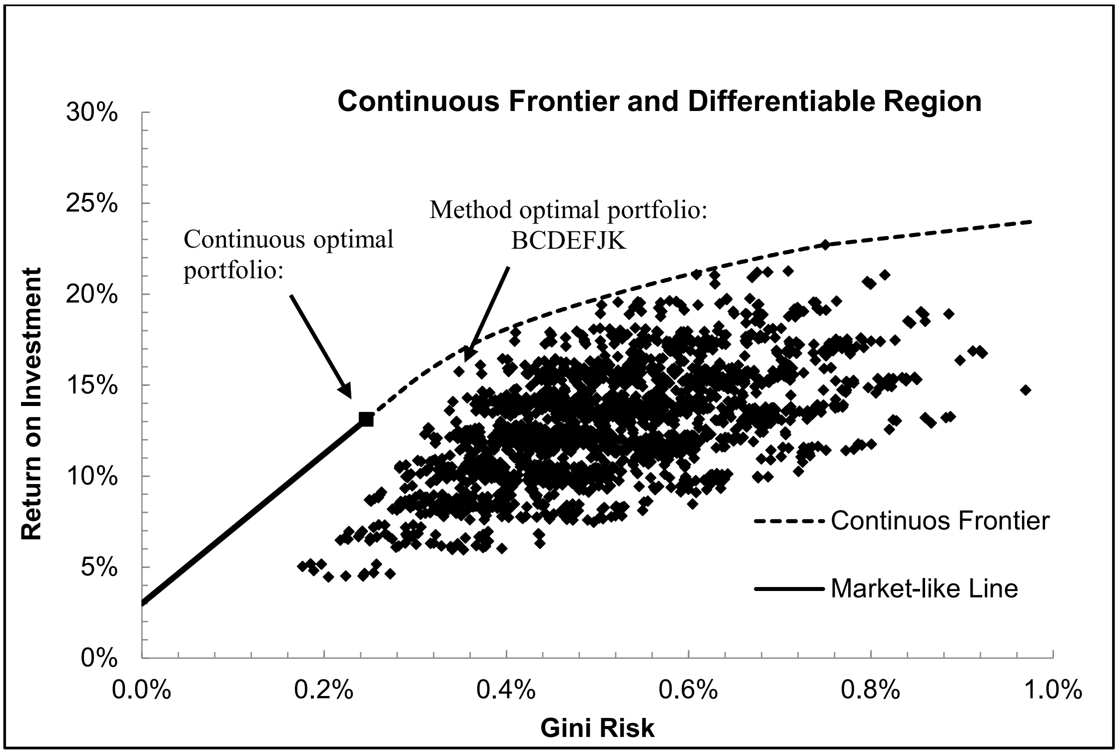

Figure 9 – Continuous frontier and differentiable region to Method II
Source: author

After GS_s calculations, we must calculate the non-diversifiable Beta-Gini risk values GB_s. The strategic use of this variable has three basic possibilities: avoiding losses when the recession is inevitable, taking advantage of the trend when growth is evident, or just being in tune with the market when the future is uncertain (we assumed the latter for the application). The formulation of variable GB_s is given by set Equation (3.23), and restrictions (3.21) and (3.22). The third among the main variables of the method is GP_s. In the CAPM and Gini-CAPM theories, its result we used to define values adjusted to the assets under study according to historical data and, in this way, highlight opportunities. The formulations for calculating GP_s are the set Equation (3.24), and restrictions (3.21) and (3.22). In the pseudocode, the calculation corresponds to steps 22, 23, and 24.

Table 5 presents the results for GS_s, GB_s and GP_s. In the first table column are the descriptions of the best-ranked portfolios according to the new method. The following three columns present the values found, respectively, for the variables Sharpe-Gini GS_s, Beta-Gini GB_s and Price-Gini GP_s. The fifth column, which determines the descending order of presentation, shows the results of the AHP multi-criteria tool. To obtain the AHP results, the small company that participated in the study determined that GS_s should have 5 times more important than GB_s, and 7 times more important than GP_s. And GB_s it should have 3 times more important than GP_s.

When processing the data before applying the AHP, the values GB_s were replaced by the modulus of their distances to the constant 1.00 (which represents a minimum non-diversifiable risk). The values were still inverted (multiplied to -1) to reverse the logic that the smaller, the better these values would be added 2 to move away from zero. In GP_s, we subtracted its values from the values of the small company under the assumption that it has relevant information in such pricing to identify opportunities. In this case, we also added 2 to move away from zero, and the smaller the difference result, the more deprecated the portfolio should be. The variable GS_s did not receive any treatment. It is important to emphasize that Table 9 shows only 2% of the portfolios ranked according to the method.

As per scope, all of these best portfolios are eligible, and we must choose the best in the rank. However, the small company decided to implement a more conservative investment, where we should apply a maximum of R$ 1,000,000.00 in projects, with the remainder applied in R_f. Thus, we cannot choose the first in the classification (BCDEFGJK), as it would require an investment above the limit. Therefore, the best two options are BCDEFJK and BCDEGJK.

Best Portfolios	Sharpe-Gini	Beta-Gini	Price-Gini	AHP Results
BCDEFJK	36.637	1.261	15.744%	0.000803
BCDEFGJK	36.317	1.473	17.893%	0.000786
BCDEGJK	35.261	1.295	16.096%	0.000776
BCDEFHJK	35.254	1.426	17.419%	0.000769
BCDEHJK	33.853	1.248	15.622%	0.000753
BCDFGJK	33.973	1.330	16.445%	0.000751
BCDEFGK	33.374	1.324	16.388%	0.000740
CDEFJK	32.518	1.098	14.096%	0.000736
ABCDEFGK	33.304	1.474	17.901%	0.000731
BCDEGHJK	32.885	1.461	17.771%	0.000724
BCDEFHK	32.199	1.277	15.915%	0.000721
BCDFJK	31.653	1.117	14.297%	0.000719
BDEFJK	31.397	1.048	13.595%	0.000718
BCDFHJK	31.901	1.283	15.972%	0.000715
ABCDEFHK	32.311	1.427	17.427%	0.000715
ABCDEFJK	32.238	1.410	17.256%	0.000715
ABCDEFK	31.693	1.261	15.752%	0.000712
BCDEFIJK	31.975	1.422	17.375%	0.000709
BCDGJK	31.186	1.152	14.648%	0.000709
ABCDEFGJK	32.514	1.623	19.405%	0.000709
CDFJK	30.697	0.954	12.649%	0.000705
ABCDEGK	31.192	1.296	16.104%	0.000701
BCDEFK	30.649	1.112	14.240%	0.000701
CDEFK	30.437	0.949	12.592%	0.000700
ABCDEGJK	31.421	1.445	17.608%	0.000698
CDEGJK	30.425	1.132	14.448%	0.000696
ABCDEFHJK	31.677	1.576	18.931%	0.000696
ABCDFGJK	31.348	1.479	17.957%	0.000695
BCDEGK	30.393	1.147	14.592%	0.000695
BCDEFGHJK	31.728	1.639	19.568%	0.000693
CDEJK	29.986	0.920	12.299%	0.000691
BCDEJK	29.971	1.083	13.947%	0.000690
ACDEFK	29.959	1.098	14.104%	0.000689
BCDEGHK	30.486	1.312	16.267%	0.000688
ABCDEGHK	30.716	1.462	17.779%	0.000684
BCDFIJK	30.096	1.279	15.928%	0.000682
BCDGHJK	30.171	1.318	16.323%	0.000682
ABCDEHK	29.968	1.249	15.630%	0.000681
ABCDEHJK	30.380	1.398	17.135%	0.000681
BCDEGIJK	30.448	1.457	17.727%	0.000679

Table 9 – Final results for selection by Method II

4.2.3 Preliminary Results Validations about Method II

Preliminary validation of the new method only at an initial level is possible by evaluating the selected portfolio (BCDEFJK) in Figure 9, which we drew from historical data. First, it is possible to note that BCDEFJK stands out in the discrete, efficient frontier. Furthermore, by the position of BCDEFJK, we can affirm that it exerts stochastic dominance over most options due to its position in the risk and return dimensions. Furthermore, in portfolio selection by the trade-off between return and risk, we desired stochastic dominance.

The graph in Figure 10 also evidences the excellence expectations to the selected portfolio. The chart shows that, unlike the other two portfolios presented (BEJ and ACDFGHIK), it is close to the line that demarcates the minimum non-diversifiable risk. In the analysis, it is also possible to state that the pricing performed by the small company for BCDEFJK has elements that lead to believe that it has a trend of ROI among the best options. Like for BEJ, which has optimistic pricing performed by the small business, unlike the option ACDFGHIK.

Both portfolios used as an example for comparison are not in the research scope strategy of seeking accordance with the portfolio similar to of market. In case BEJ is to times of evident recession, and ACDFGHIK for when the economy is growing. Nevertheless, according to the research scope, by determination of the small company applying the method, the preference should be to seek to be more correlated with the market due to the moment of future uncertainty.

It is essential to emphasize that “a complete validation of a mathematical and statistical method only happens in the long term by confirming results close to those expected and with vast replications to prove efficiency and effectiveness. Statistically, a number between 30 and 60 periods would be reasonable for evaluating a method's results, for example, submitting these results to hypothesis tests to compare them with classical, more user options, between others. However, in the absence of application results, a widely accepted theory for initial validations in portfolio selection is the Monte Carlo Simulation, where it is a possible project the probabilities of interest in stochastics ways” (MONTGOMERY, 2021).

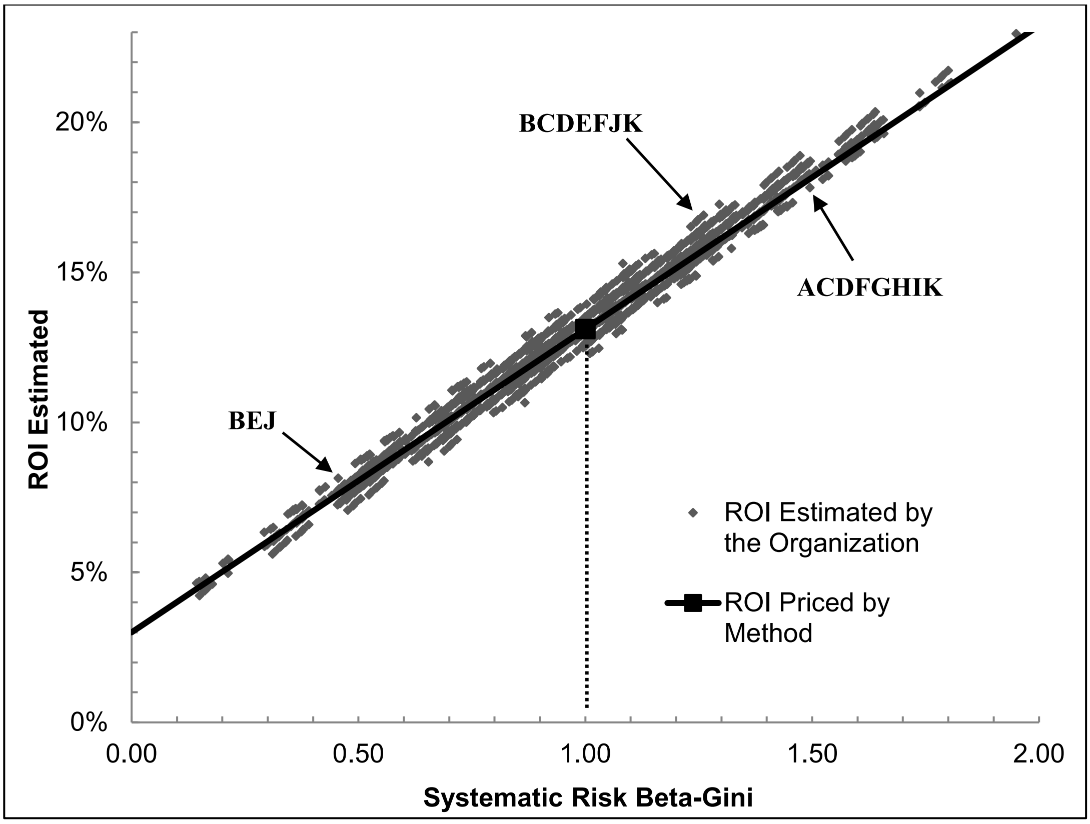

Figure 10 – Similar security market line by Method II

Source: authors

At this time, there are not enough data to prove or refute the efficiency of the new method vehemently, at least not in period extensions for statistically valid studies. Nevertheless, if there were minimum periods, the last 20 months would have significant distortions due to the current economic crisis caused by the Covid-19 pandemic. On the other hand, portfolio implementations are unnecessary for these assessments, as the main input for the analysis, adjacent energy consumption, does not depend on implementations.

Therefore, the solution to reinforce the new method's preliminary validations (based on historical data so far) is to use the Monte Carlo Simulation Theory, which is significantly accepted and applied. Thus, we generated 10^6 data for each project in the simulations according to their triangular probability distributions. Then, in sequence, we compared portfolios selected by the new method with selected by Net Present Value (NPV), Internal Rate of Return (IRR), and Pay-Back (PB). The reason is that these models are the most used contemporaneously for the selection of discrete asset portfolios, including for investment projects, which is the case observed in this research.

Respectively and in that order, we selected 15 using the NVP method, 15 using the IRR, and 14 using the PB. Also, we used an exclusive selection not to generate portfolio redundancies, and the order was according to the highest level of use of each of the three options. For the new method, we included only the two best-classified portfolios (BCDEFJK and BCDEGJK). Furthermore, as established, we applied an initial investment limit both for the new method and for its competitors of R$ 1,000,000.00.

The restriction of initial investment imposed that the portfolios had a maximum of 7 components. However, we observed that the maximum number of elements did not limit the benefit of portfolio diversification, according to the results shown in Table 10. The reason is that the portfolios have results of probabilities less than $2R_f$ and less than $3R_f$ significantly low. The levels of $2R_f$ and $3R_f$ refer to the Minimum Attractiveness Rate (MAR) possibilities stipulated by the small financing company. For values smaller than $2R_f$ the percentages were very close to 0.00 % and for values smaller than $3R_f$, they were between 0.00% and 0.50%.

Table 10 also presents in its first column the portfolio compositions in ascending alphabetical order. Its second column presents the method identification that selected the portfolio, where MT is the method proposed by this research. The third and fourth columns show fundamental criteria according to the Utility Theory: at high levels of investments, the most significant concerns are related to possible losses. The reason is that in the two columns, there are, respectively, the classification of each of the portfolios analysed according to excellence in minimum ROI probabilities for levels lower than $2R_f$ and $3R_f$. In these two criteria, the portfolios selected by the new method occupy the first and second place rankings (BCDEFJK and BCDEGJK).

The fifth column of Table 10 presents rankings according to probabilities of significantly satisfactory ROI levels. The company that is applying the method defined the criteria, where the analysed ROI probabilities were higher than $4R_f$ ((the values for all 46 portfolios were greater than 98%). In this case, the portfolios selected by the method do not have ratings as good as the observed for previous criteria, but they were still reasonably. In the criterion, the classification of BCDEFJK got fourth place, and the classification of BCDEGJK the second place.

The sixth Table 10 column presents the rankings of the portfolios according to the highest levels of Gini correlation that they have with the market optimum x_j^* the values for all 46 portfolios were between 0.44 and 0.90). We analyzed this characteristic in the simulation to determine the ability of the portfolios to behave similarly to the market. In fact, according to the CAPM and Gini-CAPM theories, portfolios with higher correlations with the market can be expected to protect investors more in uncertain scenarios or without clear prospects for both growth and recession. In this criterion, the portfolios selected by the new method again occupy first place and second place (respectively to BCDEFJK and BCDEGJK).

Portfolio	Select. Criteria	Rank ≤ $2R_f$	Rank for ≤ $3R_f$	Rank for ≥ $4R_f$	Rank for Correl.	General Rank	Without Covar.
ABCDEGK	PB	5	5	8	5	5	22
ABCDFGK	NVP	8	8	5	8	8	8
ABCDGHK	NVP	14	15	15	14	15	21
ABCDGIK	MAR	19	19	18	19	19	33
ABCDGJK	MAR	10	10	11	10	10	23
ABCFGHK	PB	31	32	35	30	32	39
ABCFGIK	PB	37	37	38	37	37	44
ACDEFGK	MAR	12	12	12	12	12	7
ACDEGHK	MAR	21	21	22	22	21	23
ACDEGIK	PB	23	23	27	23	24	34
ACDFGHK	NVP	36	36	30	36	36	9
ACDFGIK	NVP	33	30	29	34	31	15
ACDFGJK	MAR	16	16	17	16	16	10
ACDGHIK	MAR	40	40	40	40	40	35
ACDGHJK	MAR	34	34	34	33	34	27
ACDGIJK	MAR	30	31	32	29	30	37
ACFGHIK	PB	45	45	45	45	45	45
BCDEFGK	NVP	4	4	3	4	4	1
BCDEFJK	MT	1	1	4	1	1	20
BCDEGHK	MAR	6	6	6	6	6	16
BCDEGIK	MAR	13	13	14	13	13	25
BCDEGJK	MT	2	2	2	2	2	17
BCDFGHI	PB	43	43	43	42	43	43
BCDFGHK	NVP	15	14	13	15	14	5
BCDFGIK	NVP	18	17	16	20	17	11
BCDFGJK	NVP	3	3	1	3	3	4
BCDFHIK	PB	28	29	33	28	29	31
BCDGHIK	NVP	35	35	31	35	35	26
BCDGHJK	NVP	7	7	7	7	7	18
BCDGIJK	MAR	11	11	10	11	11	28
BCEFGHK	PB	22	22	26	21	22	36
BCFGHIK	MAR	41	42	42	41	41	40
BCFGHJK	PB	24	24	28	24	26	38
BCFGIJK	PB	32	33	36	32	33	41
BDFGHIK	PB	44	44	44	44	44	42
CDEFGHK	NVP	26	26	21	26	25	2
CDEFGIK	NVP	27	27	25	27	27	11
CDEFGJK	MAR	9	9	9	9	9	3
CDEGHIK	MAR	38	39	39	38	39	28
CDEGHJK	PB	17	18	19	17	17	19
CDEGIJK	PB	20	20	23	18	20	30
CDFGHIK	NVP	42	41	41	43	42	13
CDFGHJK	NVP	29	28	24	31	28	6
CDFGIJK	NVP	25	25	20	25	23	14
CDGHIJK	MAR	39	38	37	39	38	31
CFGHIJK	PB	46	46	46	46	46	46

Table 10 – Results of Monte Carlo simulations according to Method II

The seventh column of Table 10 presents a final rank for each portfolio according to the average rating to all criteria analyzed. Therefore, this column shows a final result synthesized. The BCDEFJK and BCDEGJK portfolios occupied first and second place in the study, which corroborates for statements about the excellence of portfolio selection by the new year method. The eighth column represents another crucial result for each portfolio: values found using a calculation identical to the previous column but not considering the Gini-covariation. When comparing this column's results with the one in the seventh, it is possible to observe significant differences in classification among most portfolios. These differences can lead to substantial selection errors.

Therefore, according to analyses based on historical presented and mainly on the numerical results of the simulations, it is possible to predict that the new method will select significantly excellent portfolios. In comparing its selections with selections by traditional and more used methods (NPV, IRR, and PB) contemporarily. Among the beneficial characteristics to be expected for portfolios selected by the method, the following stand out: the lowest ROI probabilities below the risk-free rate or minimum attractiveness rates, the highest probabilities of ROI attractive to investors, greater protections against market uncertainties, between others.

The method also contributes to deciding on selection portfolios with more precision, accuracy, and clarity. The reason is that, again, based on historical and numerical simulations data, using the method in the adjacent selection helps avoid mistakes caused by not considering Gini-covariations in the ROI. These covariations are significantly common, and the traditional methods do not take them into account.

5 CONCLUSIONS

Researches show many methods for selecting asset portfolios. However, some researches also indicate that the number of options significantly reduces for project portfolio selections considering the trade-off of risk and return and using a more robust metric against non-normal risk destructions. Therefore, in this research, we propose a method to select projects portfolios considering the trade-off cited and using multicriteria of CAPM, skewness, and stochastic dominance, both with the Gini coefficient as a risk metric. In developing the methods, we also seek to reduce applications' cost and time by eliminating the need for simulations, which should probably help increase the utilizations number. The method also has another beneficial feature concerning other purely economic, which are more common in these evaluations: a financial structure, which uses a more significant number of parameters to make the evaluation more accurate and lucid.

In sequence, we structured the methods selecting portfolios projects to photovoltaic solar energy generation type and small and micro-companies not just from Brazil, but from all over the world. In addition, we planned the methods they have application by other small businesses that want to invest in the finance of the projects in question. In fact, we look in this step for methods to try to benefit the various parties involved: small and micro companies that will install systems for their energy generation and that will amortize the investments in the short and medium-term, small companies that will use the method to select the possible financing investments professionally, and society in general with the technological development that support and encourages the generation of clean, sustainable and environmentally friendly energy, and with significant novelties.

According to evaluations based on historical data and, mainly, on data obtained by Monte Carlo simulation, it is possible to hope that the methods would select significantly satisfactory portfolios compared with others selected by traditional and more used models. These evaluations also show that the portfolios selected by the method have good probabilities of satisfactory returns and low probabilities of losses.

Therefore, we can conclude that we have achieved the objective of this research. However, for preliminary validation, we did not run comparisons with methods using the variance metric because the central premise of this research is that we should not use variance to analyze non-normal probability distributions. But the evaluations also show that the portfolios have reasonable probabilistic expectations and satisfactory protection to avoid mistakes for not considering covariations in return on investment. The methods also present theoretical contributions in adaptations of the benchmark models. However, the excellence of the method will only be statistically proven after evaluations of post-selection data, using an expressive periods number, based on replications in significant numbers, and when the economic recession due to the Covid-19 pandemic is not influencing plus the results.

In the research, we seek to contribute in a plural way to society, and we can highlight the main ones. Among them is the increase in the technological mastery of project portfolio selection, which is beneficial for most organizations, as most of them have in their structure adjacent activity as significantly important. The reason is that the activity, if well-executed, can help the organization in several aspects: maximize the return on capital, minimize the business risks, increase the competitive position, rationalize the allocation of resources in general, among others. Just as, if poorly executed, the activity can lead the company to bankruptcy. Another significant contribution refers to the literary gaps inherent to the research, that is, the absence of methods similar to those we developed in the course of the work. In fact, these methods were also structured with a bias for applications in small and micro companies, in addition to being biased for solar generation projects using photovoltaic cells. These last two contributions raise the research results to significantly higher levels, due to their importance to society in general.

It is also important to discard the feasibility of implementing the models developed by the research. We estimate that, for a study with approximately 20 projects, a parametric schedule is possible in 12 hours, and that changing the entries of this schedule to study other projects is possible in less than 30 minutes. But also, it is evident that the methods presented in this research have several limitations.

Among the limitations, we can highlight that in CAPM and Gini-CAPM "the assets are divisible, that is, it is possible to obtain and retain fractions of assets", but not for the methods developed by this research. Among the implications of non-validation, we must highlight the possibility of not meeting optimality conditions according to the reference models, and the impediment to the use of algorithms based on gradient variation to solutions. Another important limitation of the methods developed in this research is the definition of the market portfolio. In developing the CAPM, Sharpe proposed that the best investment option should be the one with the greatest excess per unit of risk. Although we assume this option a priori in the research, the resulting portfolio is not feasible, as its components are not discrete. In this case, among the losses, we highlight the undesirable fact that strategic compositions are not possible with the optimal portfolio of the set.

In fact, there are several other limitations to this research that are discussed in detail within the developments of each method, but there is also a latent need for future research to continue this. In a synthesized form we must emphasize that these works will be of great value, whether for the development of other methods, improving the methods proposed by this research, for application of the methods in other scenarios, for proving or refuting the expectations of our method, and also for use other latent technologies today to try to solve the same issues.

Regarding the possibility of new methods for solving research problems, certainly a very strong aspect today is regarding the update of artificial intelligence. However, the use of other technologies currently in evidence to improve the methods developed in this research is also an attractive possibility. Another interesting possibility is the application of the methods developed by this research in other scenarios, and a significantly challenging would be the stock market. But in fact, all these possibilities can help, either to prove our expectations regarding the methods developed, or to refute them.

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