

**FEDERAL UNIVERSITY OF ITAJUBÁ – UNIFEI
MECHANICAL ENGINEERING INSTITUTE
POSTGRADUATE PROGRAM IN MECHANICAL
ENGINEERING**

Luiz Alberto Monnerat de Amorim

**Study on Multi-Modal Dynamic
Vibration Absorbers Applied to
Helicopters**

Itajubá/MG

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Dissertation submitted to the Graduate Program in Mechanical Engineering, from Mechanical Engineering Institute of the Federal University of Itajubá, as a requirement to obtain the title of Master of Science in Mechanical Engineering.

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Co-supervisor: Prof. Marcelo Santiago de Sousa

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July 10th, 2026, awarding the author the title of
Master of Science in Mechanical Engineering.

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2026

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*“How can less be more?
It is impossible, more is more!”*

Yngwie Malmsteen.

RESUMO

Controle de vibrações é um desafio crítico na engenharia de helicópteros, onde vibrações estruturais de alta amplitude aceleram o desgaste e a fadiga dos componentes, ao mesmo tempo em que degradam o desempenho e o conforto da tripulação. Uma ferramenta de engenharia bastante aplicada a estes casos é o uso dos Absorvedores Dinâmicos de Vibração (ADVs) que possuem a capacidade de atenuar os níveis de vibrações nos pontos de acoplamento. Como uma evolução dos ADVs, surgem os chamados Absorvedores Dinâmicos de Vibração Multimodais (ADVMM), que podem melhorar o desempenho ao atenuar simultaneamente várias frequências (harmônicas) do rotor com um único dispositivo. Neste trabalho, é proposto um estudo acerca da implementação de uma metodologia de projeto baseada no Método dos Elementos Finitos (MEF) para ADVMMs adaptados às frequências dos rotores principal e de cauda de um helicóptero Lynx da Marinha do Brasil, utilizando molas de lâmina em fibra de carbono integradas a massas inerciais de aço. A análise experimental de vibração com dados de acelerômetros de ensaios em voo orientou a seleção das frequências-alvo, revelando que, embora as frequências de excitação dominantes sejam governadas pelas harmônicas dos rotores principal e de cauda, a distribuição relativa de amplitude varia significativamente entre as posições do piloto e do copiloto e entre as diferentes condições de voo. A partir dessa análise, quatro frequências-alvo foram selecionadas para atenuação pelo ADVMM. Uma rotina de otimização em dois estágios baseada em algoritmo genético foi implementada: o primeiro estágio alinha as frequências naturais isoladas dos ADVMMs com as frequências-alvo por meio de análise modal, e o segundo estágio refina sua geometria por meio de análise harmônica acoplada, levando em conta o amortecimento e a interação com a estrutura principal. Duas geometrias candidatas foram propostas, aproveitando múltiplos modos de vibração, como flexão e torção. A primeira geometria é composta por dois blocos de massa, cada um contribuindo com dois modos de vibração, gerando duas antirressonâncias por bloco. A segunda geometria é composta por três blocos de massa, cujo movimento combinado contribui para quatro modos de vibração da estrutura, gerando quatro antirressonâncias. Ao gerar múltiplas antirressonâncias em um único dispositivo, os ADVMMs propostos alcançam uma supressão de vibração mais abrangente do que os absorvedores convencionais de modo único, ao mesmo tempo em que fazem um uso mais eficiente do peso do absorvedor em comparação à adição de múltiplos dispositivos independentes à estrutura.

Palavras-chaves: Vibrações Mecânicas, Absorvedor Dinâmico de Vibração, Controle de Vibração Multimodal, Análise por Elementos Finitos, Algoritmo Genético.

ABSTRACT

Vibration control is a critical challenge in helicopter engineering, where high-amplitude structural vibrations accelerate component wear and fatigue while degrading crew performance and comfort. Conventional Dynamic Vibration Absorbers (DVA) are widely used in helicopters, as an evolution of the DVA, there is the Multi-Modal Vibration Absorber (MMDVA), which can suppress several rotor harmonics simultaneously with a single device. This study implements a finite element method (FEM) based design methodology for MMDVAs tailored to the main and tail rotor frequencies of a Brazilian Navy Lynx helicopter, utilizing carbon fiber leaf springs integrated with steel inertial masses. Experimental vibration analysis using flight test accelerometer data informed target frequency selection, revealing that while the dominant excitation frequencies are governed by main rotor and tail rotor harmonics, their relative amplitude distribution varies significantly between pilot and co-pilot locations and across flight conditions. From this analysis, four target frequencies were selected for each MMDVA to attenuate. A two-stage optimization routine based on a genetic algorithm was implemented: the first stage aligns the isolated natural frequencies of the MMDVAs with the target frequencies using modal analysis, and the second stage refines its geometry through coupled harmonic analysis to account for damping and interaction with the host structure. Two candidate geometries were proposed, taking advantage of multiple vibration mode shapes like flexion and torsion. The first geometry comprises two mass blocks, each contributing with two vibration modes, generating two antiresonances per block. The second geometry comprises three mass blocks, whose combined motion contributes to four vibration modes of the structure, generating four antiresonances. By generating multiple antiresonances with a single device, the proposed MMDVAs achieve broader vibration suppression than conventional single-mode absorbers, making more efficient use of absorber weight compared to adding multiple independent devices to the structure.

Key-words: Mechanical Vibrations, Dynamic Vibration Absorber, Multi-Modal Vibration Control, Finite Element Analysis, Genetic Algorithm.

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LIST OF ACRONYMS AND ABBREVIATIONS

ACSR	active control of structural response
APDL	ANSYS® Parametric Design Language
BPF	blade passage frequency
DAVI	dynamic antiresonant vibration isolator
DVA	dynamic vibration absorber
FEA	finite element analysis
FEM	finite element method
FFT	fast Fourier transform
FRF	frequency response function
GA	genetic algorithm
GEMEC	Grupo de Estudos em Mecânica Experimental e Computacional
HHC	higher harmonic control
IBC	individual harmonic control
IPEV	Brazilian Flight Test and Research Institute
IPS	inches per second
LIVE	liquid inertia vibration eliminator
MGB	main gear box
MMDVA	multi-modal dynamic vibration absorber
RMS	root mean square

LIST OF SYMBOLS

ω	angular frequency, $[\text{rad s}^{-1}]$
N_b	number of rotor blades
m_j	mass of the j -th body ($j = 1$ primary, $j \geq 2$ absorbers)
k_j	stiffness of the j -th spring
μ_j	mass ratio of the j -th absorber, m_j/m_1
c_j	viscous damping coefficient of the j -th damper
ζ_j	damping ratio of the j -th absorber
α_j	tuning ratio of the j -th absorber, $\omega_{a,j}/\omega_n$
$A(\omega)$	complex acceleration spectrum, $[\text{m s}^{-2}]$
$V(\omega)$	complex velocity spectrum, $[\text{m s}^{-1}]$
E	Young's modulus, $[\text{GPa}]$
G	shear modulus, $[\text{GPa}]$
ν	Poisson's ratio
ρ	density of the structure's material, $[\text{kg m}^{-3}]$
i	imaginary unit
$f_{n,k}$	k -th extracted natural frequency of the absorber, $[\text{Hz}]$
$f_{t,k}$	k -th target excitation frequency, $[\text{Hz}]$
$f_{p,k}$	frequency of the k -th identified FRF peak, $[\text{Hz}]$
P	penalty value for missing antiresonant modes
$\mathbf{M}$	global mass matrix
$\mathbf{C}$	global damping matrix
$\mathbf{K}$	global stiffness matrix
$\mathbf{Z}(\omega)$	dynamic impedance matrix
Φ	modal matrix, columns are mass-normalized eigenvectors
$\mathbf{N}$	shape-function matrix
$\mathbf{B}$	strain-displacement operator
$\mathbf{D}$	constitutive matrix

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1 INTRODUCTION

1.1 Motivation

Helicopters are indispensable to a wide range of civilian and military operations. Their vertical take-off and landing capability enables deployment in missions such as search and rescue, maritime patrol, medical evacuation, troop transport, shipborne operations, and anti-submarine warfare. A historical overview of helicopter engineering is presented by [Leishman \(2006\)](#).

Despite their unique operational capabilities, helicopters present high operational costs and maintenance challenges. High amplitude rotor-induced vibration is an inherent consequence of helicopter aerodynamics. According to [Bramwell, Balmford & Done \(2001\)](#), vibration control in helicopters is motivated by four main concerns: crew efficiency and safety, passenger comfort, equipment reliability, and structural fatigue life. The effects of whole-body vibration on human performance and health are further documented by [Mansfield \(2004\)](#), [Harris & Piersol \(2002, Chapter 42\)](#) and [Teixeira \(2025\)](#).

Vibration levels in helicopter cabins have decreased substantially over the past decades as new control technologies have been developed and deployed. [Welsh \(2018\)](#) reports that average cabin floor vibration in the 1960s was often around 0.3 g. Current levels lie between 0.1 g and 0.2 g throughout most of the cockpit and cabin, a significant improvement. A reasonable target for next-generation rotorcraft, according to [Welsh \(2018\)](#), is around 0.03 g to 0.05 g. These targets are rarely achieved across the full cabin in current operational aircraft, indicating that vibration control remains an active engineering problem with substantial room for improvement.

A widely used approach to mitigating these vibrations is the [dynamic vibration absorber \(DVA\)](#), first proposed by [Frahm \(1911\)](#). In its classical form, a DVA is an auxiliary spring-mass system attached to the primary structure and tuned so that its resonance coincides with the excitation frequency, producing an antiresonance in the response of the host structure at that frequency. Because rotor harmonics impose well-defined forcing frequencies on the airframe, helicopters are a natural application for tuned absorbers, and several operational rotorcraft incorporate them, examples are shown by [Bramwell, Balmford & Done \(2001\)](#), [Bielawa \(2006\)](#) and [Welsh \(2018\)](#).

A primary limitation of passive absorbers, however, is the structural mass they add to the aircraft. [Welsh \(2018\)](#) discusses this cost explicitly: each device contributes dead weight that competes directly with payload, fuel, and mission equipment, and the penalty compounds when multiple absorbers are installed to suppress multiple harmonics. Two complementary strategies have emerged to reduce this cost. The first replaces the metallic

spring components with a composite material such as carbon-fiber-reinforced polymer, which achieves equivalent stiffness at lower mass. The second is the [multi-modal dynamic vibration absorber \(MMDVA\)](#), which improves mass efficiency by consolidating multiple absorbers into one device. The present work explores the combination of these strategies: an [MMDVA](#) with composite spring components, applied to vibration suppression on an operational helicopter.

1.2 Vibration control in helicopters

Helicopter engineering and rotor dynamics are covered in depth by [Johnson \(2013\)](#), [Bielawa \(2006\)](#), [Leishman \(2006\)](#) and [Padfield \(2008\)](#). The main rotor of a conventional helicopter spins at a nearly constant angular velocity ω , held within a narrow band by the engine governor. Flight control is achieved by varying the pitch of the rotor blades rather than the rotor speed. Collective pitch changes the pitch of all blades by the same amount and sets the magnitude of rotor thrust, while cyclic pitch varies the blade pitch with azimuth and tilts the rotor disk to produce horizontal motion. During each revolution, the blade undergoes three principal motions: flapping out of the rotor plane, lead-lag in the plane of rotation, and pitching about the blade's pitch axis. Because these motions repeat every revolution, they are periodic at ω and have harmonic components at its integer multiples. The corresponding hub loads are transferred to the fuselage, and contributions from the individual blades combine such that only multiples of ω appear in the fixed airframe frame. Among all the harmonics, the [blade passage frequency \(BPF\)](#) $N_b\omega$, where N_b is the number of rotor blades, is usually the strongest excitation source.

Several technologies are used to mitigate the vibration transmitted from the rotor to the airframe. Fuselage-mounted and rotor-mounted classical Frahm-type [DVAs](#) suppress specific harmonics. Isolation systems use compliant [main gear box \(MGB\)](#) mounts to achieve low transmissibility at the frequencies of interest. Active methods, in contrast, supply energy to the structure through actuators driven by feedback control.

Fuselage-mounted [DVAs](#) act in the fixed frame, usually tuned to the $N_b\omega$ harmonic. In their classical form they consist of a spring-mass element attached to the airframe at a location of high vibration response, sized so that its natural frequency coincides with the target harmonic. The theoretical foundations of [DVA](#) tuning, together with the multi-modal extensions central to this work, are developed in Chapter 2. [Bramwell, Balmford & Done \(2001\)](#) shows the example of the Sea King's [DVA](#) which uses a battery as weight (Figure 1.1). [Welsh \(2018\)](#) illustrates the passive absorbers in the UH-60L Black Hawk (Figure 1.2).

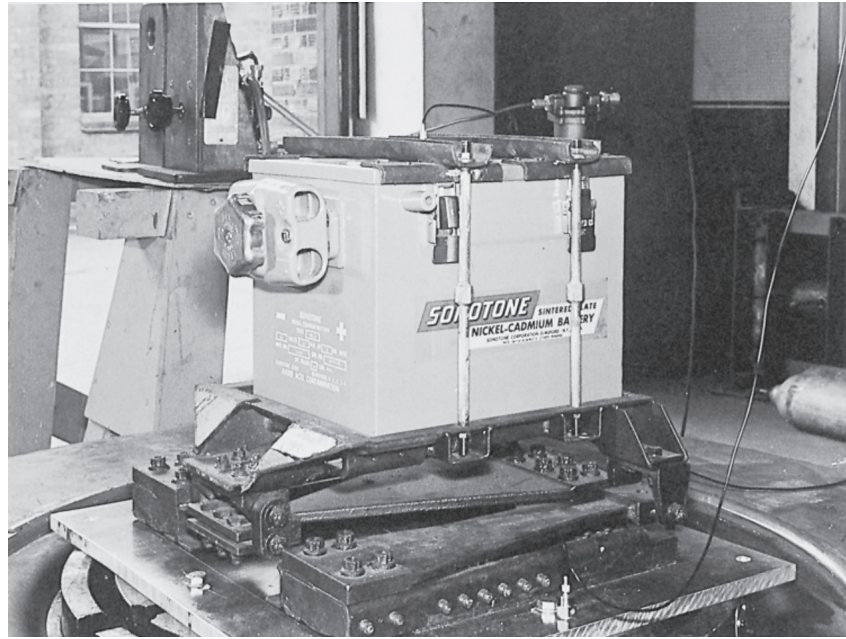


Figure 1.1 – Sea King DVA.

Source: Bramwell, Balmford & Done (2001).

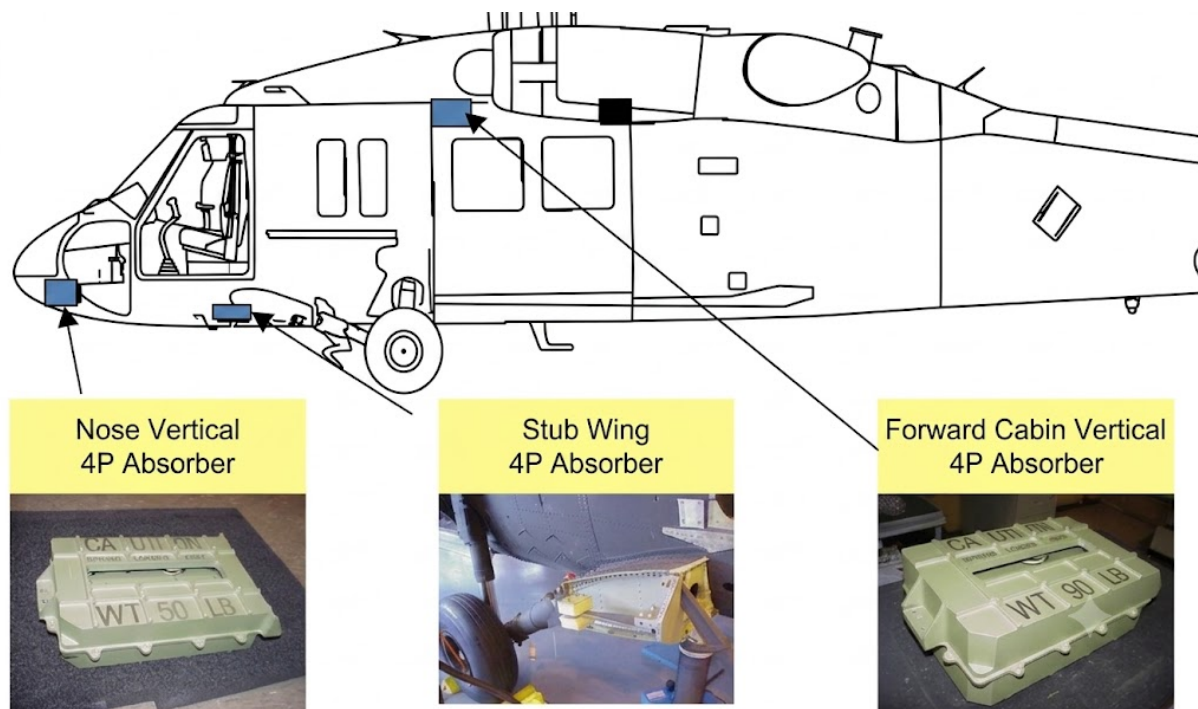


Figure 1.2 – Black Hawk fuselage DVAs.

Source: Adapted from Welsh (2018).

The equations of motion of pendular rotor-based DVAs are described by Bielawa (2006), the bifilar variation of pendular absorbers is detailed in Bramwell, Balmford & Done (2001) and Bielawa (2006). The bifilar absorber was introduced by Paul (1969) and has been used in many Sikorsky helicopters. A different rotor-based design, the flexispring

absorber, was developed for the Lynx and is described by [White \(1979\)](#). Figures 1.3 and 1.4 illustrate the pendular and bifilar absorbers respectively. Figure 1.5 shows a top view of the flexispring and Figure 1.6 displays the complete flexispring absorber, as described in [White \(1979\)](#), the lower cover, top cover, outer rings and tuning weights act as the moving mass.

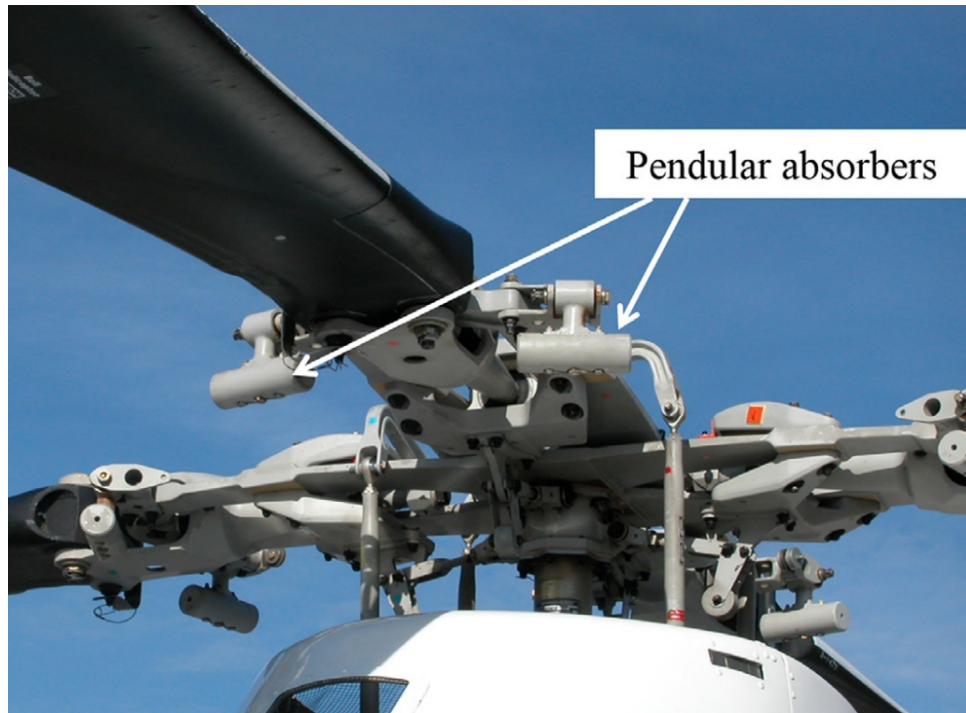
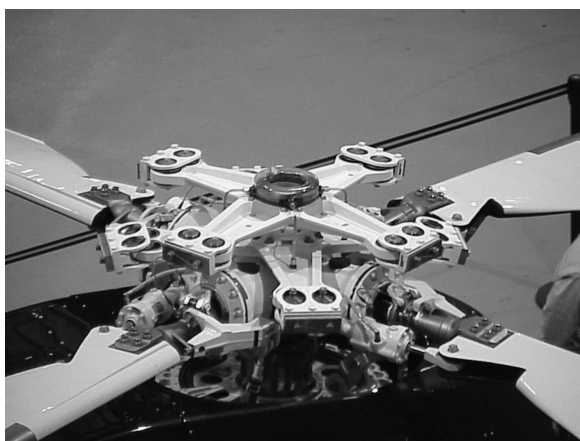
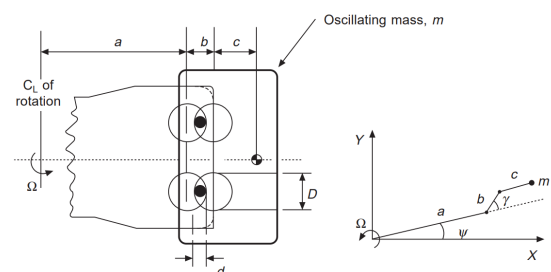


Figure 1.3 – Pendulum absorber.

Source: [Welsh \(2018\)](#).



Source: [Bielawa \(2006\)](#).



Source: [Bramwell, Balmford & Done \(2001\)](#).

Figure 1.4 – Bifilar pendulum absorber.

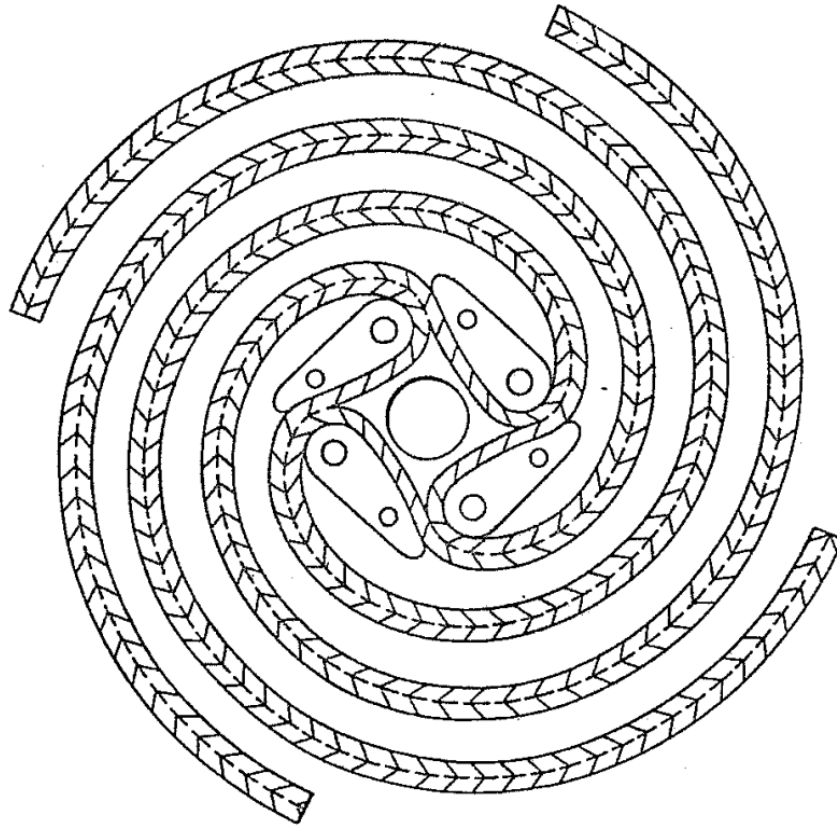


Figure 1.5 – Flexispring.

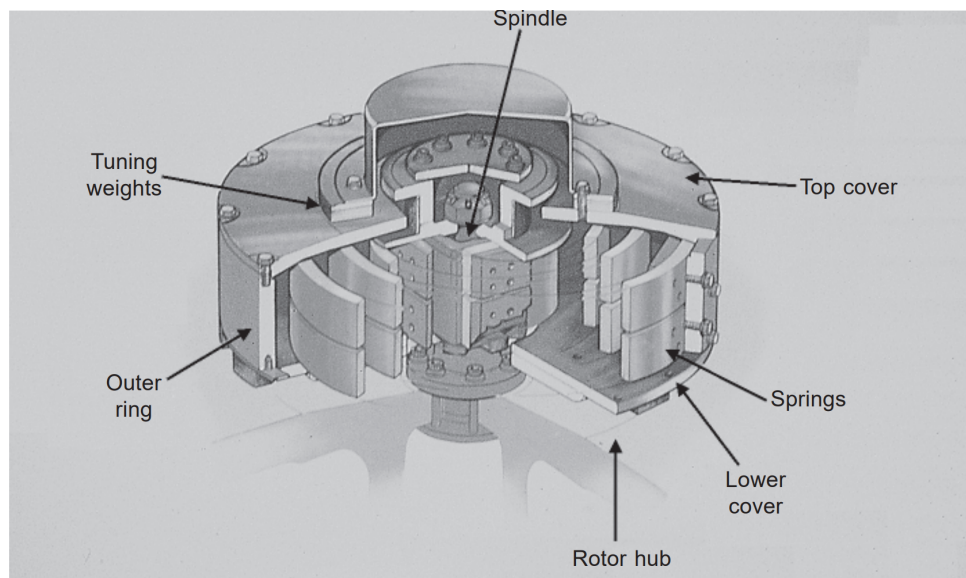
Source: [White \(1979\)](#).

Figure 1.6 – Flexispring absorber.

Source: [Bramwell, Balmford & Done \(2001\)](#).

Isolation systems take a different approach: rather than absorbing energy at a tuned frequency, they interpose compliant mounts between the [MGB](#) and the airframe to reduce the transmissibility of rotor-induced loads to the fuselage. [Flannelly & Jones \(1978\)](#)

introduced the [dynamic antiresonant vibration isolator \(DAVI\)](#), in which a small mass is mounted on a lever pivoting between the isolated body and the supporting structure. Because the lever amplifies the motion of the small mass relative to the relative displacement across the mount, its inertial force is amplified by the square of the lever ratio. At the target frequency, this amplified inertial force cancels the spring force, producing an antiresonance and near-zero transmissibility, all with a much smaller mass than a direct tuned absorber would require. [Halwes & Nicks \(1986\)](#) and [Halwes \(1987\)](#) developed this concept into the [liquid inertia vibration eliminator \(LIVE\)](#), in which the lever and mass are replaced by a column of dense fluid oscillating in a tuning passage; the fluid inertia plays the role of the amplified mass and the device isolates the vertical component of rotor-transmitted vibration between the MGB and the airframe.

Active methods of helicopter vibration control fall into two categories: those that act at the rotor to suppress hub loads at their source, and those that act on the fuselage to cancel the response after the loads have been transmitted. The first category is dominated by [higher harmonic control \(HHC\)](#), first proposed in the 1960s for a UH-1 helicopter ([WERNICKE; DREES, 1963](#) apud [WELSH, 2018](#)). [HHC](#) takes advantage of the hydraulic actuators that drive the swashplate for flight control, superimposing oscillatory inputs on the collective and cyclic commands so as to cancel the harmonic blade airloads that produce the hub forces. [Individual harmonic control \(IBC\)](#), proposed by [Ham \(1986\)](#), removes the swashplate constraint and actuates each blade independently in the rotating frame, allowing a richer set of pitch inputs than [HHC](#) can produce at the cost of dedicated per-blade actuators. Active trailing-edge flaps and active-twist rotors are later [IBC](#) implementations in which the pitch authority comes from a flap deflection or distributed blade torsion rather than from a root actuator. The fuselage-based category is represented by [active control of structural response \(ACSR\)](#), described in [Cribbs & Friedmann \(1999\)](#), in which actuators mounted at selected airframe locations are driven by accelerometer feedback to produce counter-forces that attenuate response at the cabin.

1.3 Dynamic Vibration Absorbers literature review

The active and semi-active solutions reviewed thus far represent the contemporary alternatives to passive vibration control. Within the passive route, the dynamic vibration absorber has remained a subject of active research, and the body of work relevant to the present study can be organized around three concerns: the extension of the absorber concept to multiple modes, the refinement of optimization procedures for selecting absorber parameters, and the application of these devices to rotorcraft.

The single-degree-of-freedom configuration introduced by [Frahm \(1911\)](#) and analyzed in detail by Den Hartog ([1947](#)) established the principle by which an auxiliary mass-spring system attached to a primary structure produces an antiresonance at a chosen

frequency. Den Hartog further derived the optimal damping of the absorber through the fixed-point method, yielding the equal-peak tuning that minimizes the maximum response amplitude of the primary structure, and the approach is now standard in the reference literature on mechanical vibration, as treated in [Rao & Yap \(1995\)](#). [Warburton \(1982\)](#) subsequently extended the optimal-parameter formulas to broader combinations of excitation and response. [Kotinda \(2005\)](#) presents numerical and experimental studies of classical single-mode DVAs.

The chosen tuning frequency may coincide with a resonance of the host structure, in which case the absorber attenuates the response amplified by that resonance, or with the frequency of a known harmonic excitation, in which case the absorber attenuates the response at the forcing frequency regardless of its position relative to the structure's modes. The two cases share the same underlying mechanism but differ in what determines the tuning target. These concepts are detailed further in Section 2.1.

An MMDVA addresses several modes of the primary system through resonators, each tuned to a distinct natural frequency, acting in concert on a shared host structure. A related approach achieves multimodal attenuation through a single distributed absorber rather than multiple discrete resonators, as in the beam-like absorbers studied by [Snowdon & Nobile \(1977\)](#) and the cruciform configurations later proposed by [Snowdon, Wolfe & Kerlin \(1984\)](#). [Cunha Jr. and Rade \(1999\)](#) also implemented beam structures as DVAs and genetic algorithm (GA) optimization to tune antiresonances. The formulation of multiple tuned mass dampers acting on continuous primary systems was established by [Igusa & Xu \(1994\)](#). [Raze & Kerschen \(2021\)](#) extended Den Hartog's equal-peak method to the multimodal case, proposing an all-equal-peak design in which the parameters of multiple tuned mass dampers are optimized so that the controlled resonance peaks are equalized in amplitude. [Wong *et al.* \(2007\)](#) treated the MMDVA design for beams under point and distributed loading, proposing a combination of a translational and a rotational absorber for beam vibration control. [Cunha Jr. \(1999\)](#) presents a study about the use of MMDVAs applied to multi-degree-of-freedom systems. [Mendez *et al.* \(2016\)](#) applied high fidelity finite element models coupled with optimization algorithms to design an MMDVA that attenuates response at multiple natural frequencies of complex structures, and [Delfino *et al.* \(2020\)](#) extended the approach to wing structures.

The selection of absorber parameters from prescribed performance objectives has been treated as an optimization problem in several formulations. [Rade & Steffen \(2000\)](#) proposed a methodology based on the substructure coupling of frequency response functions (FRFs), expressing the response of the composite system as a function of the absorber parameters and optimizing it over a target frequency band. [Hadi & Arfiadi \(1998\)](#) applied genetic algorithms to the design of tuned mass dampers for multi-degree-of-freedom structures and reported that the procedure determines the optimal parameters without prior specification of the modes to be controlled. [Leung & Zhang \(2009\)](#) used particle

swarm optimization for the same class of problem under several excitation types, including white-noise force and base acceleration, and provided explicit expressions for use in design.

The application of dynamic vibration absorbers to helicopters differs from the structural cases above in the nature of the target. The dominant vibration on a helicopter airframe is the harmonic excitation transmitted from the main rotor at integer multiples of the blade-passing frequency, and the absorbers reviewed in section 1.2 are tuned to these excitation frequencies rather than to airframe modes. Beyond the description of individual devices, comparative evaluation of competing solutions on rotorcraft platforms has received attention. Tamer *et al.* (2019) examined a range of passive, semi-active, and active vibration reduction devices within a unified evaluation framework, assessing their effect on ride quality at multiple cabin locations and identifying the trade-offs inherent to airframes with multiple load paths. The use of composite materials as the spring component of a helicopter absorber appears in White (1979), where a glass fiber-reinforced plastic element provides the stiffness of a fixed-frequency rotor-head absorber.

The body of work surveyed indicates three established directions. MMDVA configurations have demonstrated attenuation across multiple modes in laboratory and simulated settings, applied to beams, plates, and wing structures, with the tuning targets coinciding with resonances of the host structure. Optimization procedures for absorber parameters have matured through frequency-band, analytical, and metaheuristic formulations. Helicopter applications have relied predominantly on single-mode passive devices with metallic spring elements, tuned to a single rotor-induced excitation frequency, and comparative evaluations have been confined to existing device configurations. The combination of a multi-modal absorber with composite spring components, addressing several rotor-induced excitation frequencies through resonators tuned by a procedure that accounts for both modal alignment and coupled harmonic response, and validated against flight measurements on a helicopter platform, has not been reported.

1.4 Research gap and objectives

The literature surveyed in the previous section establishes that multi-modal dynamic vibration absorbers have demonstrated attenuation across several resonances of host structures, that optimization procedures for absorber parameters have matured through analytical and metaheuristic formulations, and that helicopter applications have nonetheless relied on single-mode passive devices and mostly with metallic spring elements, tuned to a single rotor-induced excitation frequency. The configuration adopted on a given platform reflects the vibration environment of that aircraft: the Sikorsky UH-60 Black Hawk combines rotor-based and fuselage-based passive absorbers (Figure 1.2), while the Westland Lynx, of which the AH-11B is the Brazilian Navy variant, relies on the rotor-head

flexispring absorber alone (Figure 1.6). The harmonic content transmitted by a helicopter main rotor is not confined to a single frequency: integer multiples of the blade-passing frequency appear in the airframe response, and a single-mode device leaves the higher harmonics unattenuated.

The present work addresses this gap through the main objective of designing an **MMDVA** with composite spring components for the Lynx helicopter, optimized to attenuate several rotor-induced harmonics simultaneously, to be achieved through the following specific objectives:

1. Characterize the harmonic content of the airframe vibration on the AH-11B from accelerometer flight data, identifying the rotor-induced frequencies at which a multi-modal absorber would provide attenuation beyond what a single-mode device can deliver;
2. Propose two candidate absorber geometries employing carbon fiber spring components, defined by a set of geometric parameters that govern their natural frequencies and modal couplings;
3. Develop a finite element model of each candidate geometry that reproduces the modal behavior governing the absorber's response over the frequency range of interest;
4. Align the natural frequencies of the absorber with the targeted rotor-induced harmonics through a first optimization stage based on modal analysis of the finite element model, treating the geometric parameters as design variables;
5. Refine the antiresonant response of the absorber through a second optimization stage in which the device is simulated coupled to a generic host beam, accounting for damping, and the geometric parameters are adjusted to shape the frequency response functions at the targeted harmonics.

1.5 Contributions

This work contributes to the literature on passive helicopter vibration control in four aspects:

1. The two-stage genetic algorithm proposed here separates modal alignment from harmonic refinement, allowing the natural frequencies of the absorber and the anti-resonant response under damping to be addressed as distinct optimization problems that share a common design space;
2. The multi-modal strategy: rather than assembling several single-mode resonators, the absorber engages multiple rotor-induced harmonics through distinct mode shapes of a single structure, including flexural modes in two orthogonal planes and torsional modes, with the geometric parameters of one device governing all targeted frequencies simultaneously;
3. The use of carbon fiber spring components in this configuration, extending to the multi-modal case a material choice previously demonstrated only in single-mode rotor-head devices;
4. The use of accelerometer flight data from the AH-11B both as the source of the rotor-induced harmonics that define the optimization targets and as the empirical basis from which the multi-modal need is established, anchoring the design to the vibration environment of the platform rather than to assumed excitation content.

1.6 Dissertation organization

The remainder of the dissertation is organized as follows. Chapter 2 reviews the theoretical foundations on which the work rests, covering dynamic vibration absorber theory, modal and harmonic finite element analysis, and genetic algorithm optimization. Chapter 3 presents the experimental characterization of the AH-11B vibration environment from accelerometer flight data, including the acquisition procedure, the signal processing, and the discussion of the results that establish the rotor-induced harmonic content. Chapter 4 describes the design methodology, detailing the two candidate geometries, the finite element modeling, and the two-stage optimization strategy. Chapter 5 presents and discusses the results of both optimization stages, the modal alignment and the harmonic refinement. Chapter 6 closes with the conclusions and directions for further work.

2 THEORETICAL REVIEW

2.1 Dynamic vibration absorber theory

The dynamic vibration absorber, in its classical form, is a two-degree-of-freedom system in which an auxiliary mass-spring-damper device is attached to a primary structure with the purpose of producing an antiresonance at a frequency of interest. The treatment that follows introduces the undamped and damped configurations of the single absorber, discusses how the idealized lumped-parameter model relates to the finite-element representation used in the remainder of this work, and extends the theory to the multi-modal case.

2.1.1 Undamped single absorber

Consider a primary system of mass m_1 and stiffness k_1 , subjected to a harmonic force of amplitude F_0 and frequency ω , to which an auxiliary mass m_2 is attached through a spring of stiffness k_2 , as shown in Figure 2.1. The equations of motion of the two-degree-of-freedom system are shown in Equation 2.1:

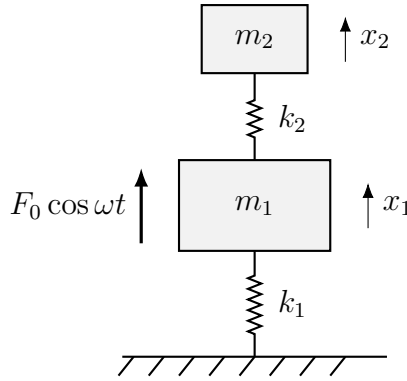


Figure 2.1 – Undamped two-degree-of-freedom system representing the classical dynamic vibration absorber.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \cos \omega t \\ 0 \end{Bmatrix}, \quad (2.1)$$

where x_1 and x_2 denote the displacements of the primary and auxiliary masses, respectively. Assuming harmonic solutions of the form $x_1(t) = X_1 \cos \omega t$ and $x_2(t) = X_2 \cos \omega t$ and solving for the steady-state amplitude of the primary mass yields

$$\frac{X}{X_{st}} = \frac{1 - (\omega/\omega_a)^2}{[1 + \mu(\omega_a/\omega_n)^2 - (\omega/\omega_n)^2][1 - (\omega/\omega_a)^2] - \mu(\omega_a/\omega_n)^2}, \quad (2.2)$$

where $X_{st} = F_0/k_1$ is the static deflection of the primary mass, $\omega_n = \sqrt{k_1/m_1}$ is the natural frequency of the primary system, $\omega_a = \sqrt{k_2/m_2}$ is the natural frequency of the absorber, and $\mu = m_2/m_1$ is the mass ratio (RAO; YAP, 1995).

Equation 2.2 shows that the response of the primary mass vanishes when $\omega = \omega_a$, regardless of the relationship between ω_a and the natural frequency of the primary system. The antiresonance is set by the absorber alone, and its location in the spectrum is determined by the choice of the tuning ratio ω_a/ω_n . Two cases illustrate the consequences of this choice.

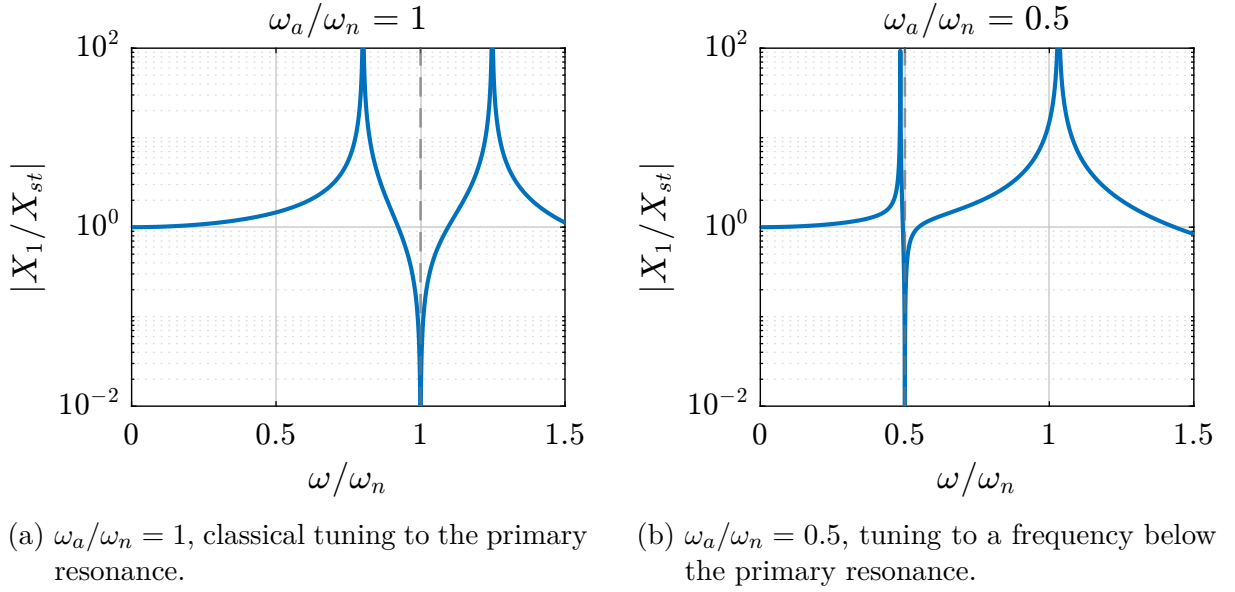


Figure 2.2 – FRF of the primary mass for the undamped absorber under two tuning conditions, mass ratio $\mu = 0.2$. The antiresonance is located at the absorber's natural frequency in both cases, but its position relative to the primary resonance differs.

In the classical case, the absorber is tuned to the natural frequency of the primary system, $\omega_a/\omega_n = 1$. The antiresonance is then placed at the primary resonance, suppressing the response that would otherwise be amplified by it. Figure 2.2 (a) shows the resulting FRF: the original resonance at $\omega/\omega_n = 1$ is replaced by an antiresonance, flanked by two new resonances at frequencies that depend on the mass ratio.

In the alternative case considered here, the absorber is tuned to a frequency below the primary natural frequency, with $\omega_a/\omega_n = 0.5$. The antiresonance is now placed at $\omega/\omega_n = 0.5$ and the primary resonance moves to a value slightly higher than $\omega/\omega_n = 1$, as shown in Figure 2.2 (b). There is also a new resonance introduced by the absorber to the left of the antiresonance.

The second case is the configuration relevant to the present work. In a helicopter airframe, the dominant excitation is the harmonic content transmitted from the main rotor, and the absorber is tuned to one of these rotor-induced frequencies rather than to a natural frequency of the airframe itself. The antiresonance is placed at the excitation

frequency, where the primary system may not exhibit a resonant peak at all, and the attenuation is achieved at that frequency regardless of the modal structure of the airframe in its vicinity.

2.1.2 Damped single absorber

The unbounded amplitudes at the two resonances flanking the antiresonance, and the exact cancellation of the primary response at the tuning frequency, are consequences of the lossless idealization. No physical system realizes these limits: every spring dissipates a fraction of the strain energy stored in it through internal friction, joint interfaces, and material hysteresis, in both the primary system and the auxiliary device that the absorber attaches to it. The viscous model, in which the dissipative force on each element is proportional to its relative velocity, is adopted here for analytical convenience; its identification with the structural damping of real materials is approximate, but the qualitative behaviour it predicts carries over to other damping representations.

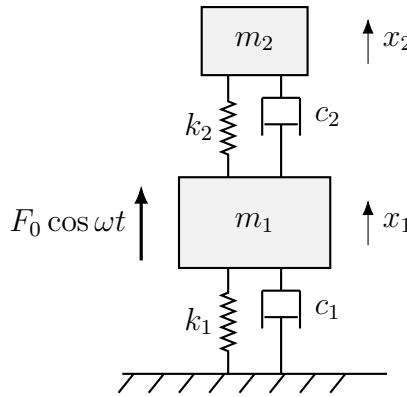


Figure 2.3 – Two-degree-of-freedom system with viscous dampers c in parallel with springs k_1 and k_2 , respectively.

Introducing viscous coefficients c_1 and c_2 in parallel with the primary and absorber springs respectively, as shown in Figure 2.3, modifies the equations of motion of the two-degree-of-freedom system to:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \cos \omega t \\ 0 \end{Bmatrix}. \quad (2.3)$$

The dimensional damping coefficients combine with the masses and stiffnesses into two damping ratios

$$\zeta_1 = \frac{c_1}{2m_1\omega_n}, \quad \zeta_2 = \frac{c_2}{2m_2\omega_a}, \quad (2.4)$$

each measuring its viscous coefficient against the critical value of the subsystem to which it

belongs. The harmonic response of the primary mass then depends on four dimensionless groups: the mass ratio μ , the tuning ratio $\alpha = \omega_a/\omega_n$, and the two damping ratios ζ_1 and ζ_2 . This is the form in which the finite-element harmonic analysis of Section 2.2.2 specifies damping, and the form in which optimal-tuning results in the literature are reported.

Applying the solution methodology presented in Rao & Yap (1995), harmonic solutions $x_1(t) = \text{Re}\{X_1 e^{i\omega t}\}$ and $x_2(t) = \text{Re}\{X_2 e^{i\omega t}\}$ are assumed and substituted into Equation 2.3, yielding a complex linear system whose solution for the steady-state amplitude of the primary mass is presented in Equation 2.5:

$$\frac{X_1}{X_{st}} = \frac{(\alpha^2 - r^2) + i 2\alpha r \zeta_2}{[(1 + \mu\alpha^2 - r^2) + i 2r(\zeta_1 + \mu\alpha\zeta_2)][(\alpha^2 - r^2) + i 2\alpha r \zeta_2] - \mu(\alpha^2 + i 2\alpha r \zeta_2)^2}, \quad (2.5)$$

where $r = \omega/\omega_n$ is the normalized excitation frequency and $X_{st} = F_0/k_1$ is the static deflection of the primary mass. The lossless limit $\zeta_1 = \zeta_2 = 0$ recovers Equation 2.2 of the previous subsection. For any $\zeta_2 > 0$ the numerator of Equation 2.5 no longer vanishes at any real frequency, so the antiresonance becomes a finite-depth minimum rather than a true zero; for any ζ_1 or ζ_2 strictly positive the denominator no longer admits real roots, so the flanking resonances become finite peaks rather than singularities.

Damping affects the frequency response of the primary mass in three respects, each visible in Figure 2.4. The flanking resonance peaks acquire finite amplitude: the lossless divergence at the natural frequencies of the coupled system is replaced by peaks of bounded height, with the height decreasing as the damping ratios grow. The antiresonance loses depth: in the lossless case the absorber spring force at $\omega = \omega_a$ exactly cancels the excitation applied to the primary mass, producing a zero of the FRF; with absorber damping, the spring force lags the displacement of the absorber mass and the cancellation becomes imperfect, so a non-zero residual response remains at the tuning frequency. The locations of the resonance peaks and of the antiresonance shift from their lossless values; for the damping levels of interest here the shifts are small and the qualitative shape of the FRF is preserved.

Figure 2.4 shows the effect of damping on the two tuning cases of Section 2.1.1, with the mass ratio held at $\mu = 0.2$ and a common damping ratio $\zeta = \zeta_1 = \zeta_2$ swept across three representative values. The choice $\zeta_1 = \zeta_2$ is a simplification that treats the structural damping of the primary and absorber elements as comparable; the qualitative conclusions below do not depend on the ratio of ζ_1 to ζ_2 so long as both are positive and of similar order.

In the classical tuning case (Figure 2.4 (a)), light damping ($\zeta = 0.02$) bounds the two flanking peaks at amplitudes a few times X_{st} without appreciably affecting the depth of the antiresonance at $r = 1$; moderate damping ($\zeta = 0.10$) attenuates the peaks further and lifts the antiresonance to a value of order X_{st} , so that the broadband response across $0.7 < r < 1.3$ is flattened at the expense of the deep notch at the tuning frequency.

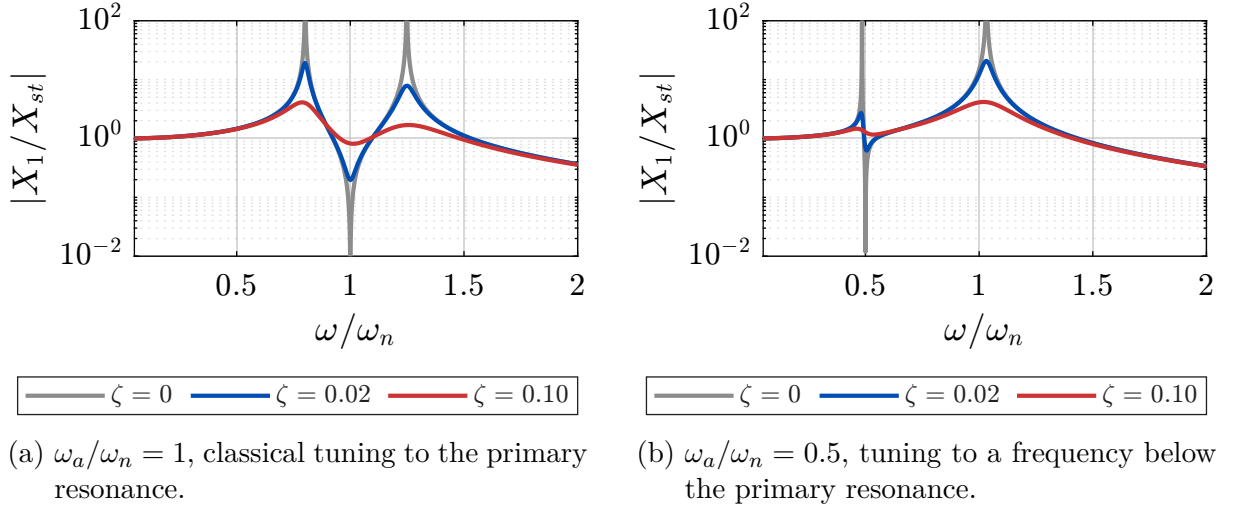


Figure 2.4 – FRF of the primary mass for the damped absorber under two tuning conditions, mass ratio $\mu = 0.2$, common damping ratio $\zeta = \zeta_1 = \zeta_2$ at three representative levels.

In the low-tuning case (Figure 2.4 (b)), the two resonances of the coupled system flank the antiresonance at $r = 0.5$ asymmetrically: one lies just below the tuning frequency, close enough to the antiresonance that the two features form a sharp dip-peak pair in the left flank, and the other lies slightly above $r = 1$, close to the frequency at which the primary system alone would resonate.

For the classical tuning case, Den Hartog (1947) treated the limiting configuration in which the primary system carries no structural damping ($\zeta_1 = 0$) and the absorber damping is realized by a dedicated viscous dashpot whose coefficient is treated as a free design parameter, distinct in role from the inherent structural damping considered in the present subsection. In this configuration the value of ζ_2 and the tuning ratio that minimize the maximum response of the primary mass admit closed-form expressions, obtained through a fixed-point argument on the frequency response function. These expressions provide the standard analytical reference for the classical absorber problem. The low-tuning case admits no analogous closed-form optimum: the absorber and the primary resonance are decoupled in frequency, and the design problem is one of placing the antiresonance at the excitation frequency while limiting the resonance the absorber introduces nearby. This is the configuration relevant to the present work.

The closed-form result of Equation 2.5 applies to the lumped-parameter representation in which the absorber spring is massless and the auxiliary mass a point. The departure of a physical absorber from this idealization, and the role of the finite element representation in capturing that departure, are the subject of the Section 2.2.

2.1.3 Multi-modal extension

The single absorber places an antiresonance at one frequency. When the response of the primary structure must be attenuated at several frequencies simultaneously, either because the excitation contains discrete harmonic components at well-separated frequencies or because more than one mode of the primary falls in the band of interest, the direct extension is to attach several absorbers in parallel, each tuned to one of the target frequencies. The configuration is illustrated here for the simplest case of two absorbers attached to a single-degree-of-freedom primary, known in the literature as the multi-tuned-mass-damper configuration as seen in [Igusa & Xu \(1994\)](#); the results extend term-by-term to any number of absorbers.

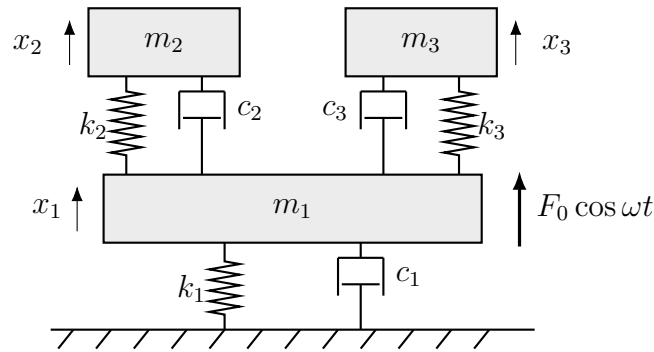


Figure 2.5 – Three-degree-of-freedom system: primary mass m_1 with structural damping, and two auxiliary masses m_2 and m_3 attached in parallel through independent spring-damper pairs.

Let m_2, k_2, c_2 and m_3, k_3, c_3 denote the masses, stiffnesses, and damping coefficients of the two absorbers, with displacements x_2 and x_3 ; the primary system retains the notation of the previous subsections. Both absorbers are attached to the primary mass through independent spring-damper pairs and are coupled to each other only through the motion of the primary, as shown in Figure 2.5. The equations of motion of the three-degree-of-freedom system are

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(t), \quad (2.6)$$

with mass, damping, and stiffness matrices

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \quad (2.7a)$$

$$\mathbf{C} = \begin{bmatrix} c_1 + c_2 + c_3 & -c_2 & -c_3 \\ -c_2 & c_2 & 0 \\ -c_3 & 0 & c_3 \end{bmatrix}, \quad (2.7b)$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 + k_3 & -k_2 & -k_3 \\ -k_2 & k_2 & 0 \\ -k_3 & 0 & k_3 \end{bmatrix}. \quad (2.7c)$$

Displacement vector $\mathbf{x} = \{x_1, x_2, x_3\}^T$, and force vector $\mathbf{F}(t) = \{F_0 \cos \omega t, 0, 0\}^T$. The harmonic response of the primary mass is obtained by assuming $\mathbf{x}(t) = \text{Re}\{\mathbf{X} e^{i\omega t}\}$ and solving the complex linear system $\mathbf{Z}(\omega) \mathbf{X} = \mathbf{F}_0$, where $\mathbf{Z}(\omega) = -\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K}$ is the impedance matrix and $\mathbf{F}_0 = \{F_0, 0, 0\}^T$. Each absorber is characterized by its mass ratio $\mu_j = m_j/m_1$, tuning ratio $\alpha_j = \omega_{a,j}/\omega_n$ with $\omega_{a,j} = \sqrt{k_j/m_j}$, and damping ratio $\zeta_j = c_j/(2m_j\omega_{a,j})$, for $j = 2, 3$.

Figure 2.6 shows the response of the primary mass for the configuration $\alpha_2 = 0.4$, $\alpha_3 = 0.6$, $\mu_2 = \mu_3 = 0.1$, at three values of a common damping ratio $\zeta = \zeta_1 = \zeta_2 = \zeta_3$. Two antiresonances appear at the tuning frequencies $r = 0.4$ and $r = 0.6$, each immediately preceded by a sharp resonance peak, and the surviving primary resonance near $r = 1$ remains essentially unaffected because both absorbers are tuned well below it. Light damping bounds the peaks to finite amplitude while preserving the depth of the two antiresonances at the tuning frequencies. Moderate damping reduces the peaks further and lifts the antiresonance floors. The qualitative shape of this response is recovered by the finite-element analyses of Chapter 5 for the MMDVA configurations used in the present work.

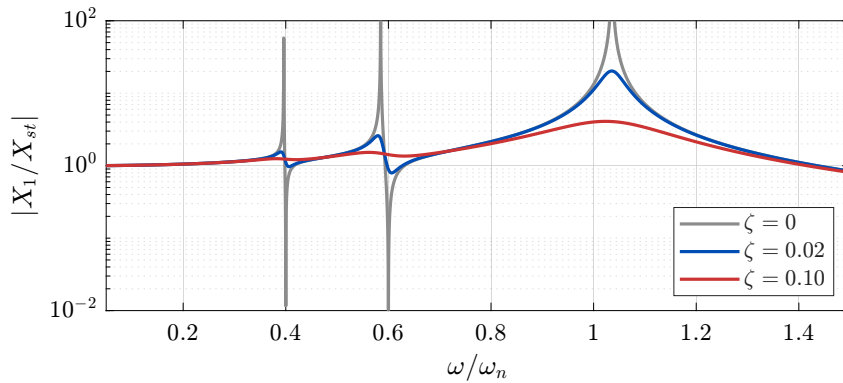


Figure 2.6 – FRF of the primary mass for the two-absorber configuration with $\alpha_2 = 0.4$, $\alpha_3 = 0.6$, $\mu_2 = \mu_3 = 0.1$, at three values of the common damping ratio $\zeta = \zeta_1 = \zeta_2 = \zeta_3$.

2.1.4 From lumped to distributed properties

The lumped parameter model assumes that the absorber consists of a massless spring and a point mass. A physical absorber departs from this idealization in two respects: the spring component has mass distributed along its length, and the auxiliary mass has finite dimensions and non-negligible mass moment of inertia. The finite element representation, which captures both effects by distributing mass and elasticity throughout the geometry of the device, produces a model with as many degrees of freedom as the discretization admits. The contrast between lumped parameter and finite element representations of the same mechanical system is treated in detail by [Hatch \(2000\)](#).

The first consequence of the distributed representation is the appearance of higher modes that the lumped model omits. A single lumped absorber has one natural frequency, given by $\omega_a = \sqrt{k/m}$; the corresponding finite-element model has the same fundamental frequency, to within discretization tolerance, but also possesses a sequence of higher modes associated with the spring component and the geometry of the auxiliary mass. Whether these higher modes interfere with the device's behavior at the targeted frequency depends on their separation from the band of interest. When the higher modes lie well above the targeted frequency, the lumped model predicts the absorber's response accurately; when they lie within or near the targeted band, the lumped model is no longer sufficient and the finite element model is required.

The second consequence is that the mass of the spring component contributes to the effective inertia of the absorber. The lumped model recovers exactly in the limit of vanishing spring mass; the practical condition is that the spring mass be small compared to the auxiliary mass, in which case the finite element fundamental frequency approaches the lumped ω_a and the lumped formulas of Section 2.1.2 apply. This condition also bears on material selection: low-density spring materials concentrate the absorber's inertia in the auxiliary mass, where it contributes to the target dynamics, rather than in the spring component.

The departure of the finite element representation from the lumped ideal is not, in itself, a defect to be minimized. When the geometry of the absorber is chosen so that several of its modes coincide with several excitation frequencies of interest, the higher modes that the lumped model omits become the basis of a multi-modal design. This is the strategy adopted in the present work, and the theoretical extension that supports it is the subject of Sections 2.2

2.2 Finite Element Method

The [finite element method \(FEM\)](#) provides numerical solutions for the equations that govern the behaviour of continuous bodies by subdividing the domain of interest into a finite number of subregions, called elements, connected at a discrete set of nodes.

This method is explained in detail by [Zienkiewicz, Taylor & Zhu \(2013\)](#) and [Bathe \(2014\)](#). In contrast with lumped parameter models, in which the inertial and elastic properties of a system are condensed into a discrete set of points connected by massless springs and inertia-less constraints, FEM preserves the distributed nature of the physical properties through the interpolation of nodal quantities by means of shape functions $\mathbf{N}(\mathbf{x})$. The displacement field within an element is approximated as

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{N}(\mathbf{x}) \mathbf{u}^{(e)}(t), \quad (2.8)$$

where $\mathbf{u}^{(e)}(t)$ collects the time-dependent nodal displacements. Application of the principle of virtual work (or any equivalent variational statement) yields the element stiffness and consistent mass matrices

$$\mathbf{K}^{(e)} = \int_{V_e} \mathbf{B}^T \mathbf{D} \mathbf{B} dV, \quad (2.9)$$

$$\mathbf{M}^{(e)} = \int_{V_e} \rho \mathbf{N}^T \mathbf{N} dV, \quad (2.10)$$

where $\mathbf{B}$ is the strain–displacement operator derived from $\mathbf{N}$, $\mathbf{D}$ is the constitutive matrix and ρ is the mass density. Assembly of the element contributions produces the global semi-discrete equation of motion,

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{C}]\{\dot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}(t)\}, \quad (2.11)$$

in which the mass and stiffness remain distributed in the same sense as in the underlying continuum: every degree of freedom participates in both inertial and elastic effects through coupled, populated matrices, as opposed to the diagonal or sparsely banded structure characteristic of lumped abstractions.

In the present work, two element formulation technologies of the ANSYS[®] Mechanical library ([ANSYS, INC., 2023a](#); [ANSYS, INC., 2023b](#)) are employed: SOLID186 is used to discretise the regions made of isotropic materials, whereas SOLSH190 is reserved for the composite components. SOLID186 is a higher-order three-dimensional 20-node structural solid element exhibiting quadratic displacement behaviour. It admits irregular meshes (with collapsed tetrahedral, pyramidal and prismatic degenerations) and is particularly well suited to the analysis of isotropic solid components.

SOLSH190 is an 8-node element formulated to bridge the gap between conventional shell and solid elements: it has solid-element connectivity (three translational degrees of freedom per node and no rotational ones) while retaining the kinematic assumptions of a shear-deformable shell. Internally, the element is based on a first-order shear-deformation kinematics consistent with the Mindlin–Reissner theory of plates and shells developed in [Mindlin \(1951\)](#) and [Reissner \(1945\)](#), which generalizes classical Kirchhoff–Love theory by relaxing the normality assumption and admitting transverse shear deformations, thus extending the applicability of shell-type formulations to moderately thick configurations.

These governing equations are also presented with detail in [Reddy \(2006\)](#). SOLSH190 also adopts a parabolic enhanced transverse-shear strain formulation, which enriches the through-thickness shear strain field beyond the constant distribution assumed by first-order shear deformation theory. Another feature exploited in this work is its native support for layered section definitions: stacking sequences, individual ply thicknesses, orientations and orthotropic material properties can be assigned through a single section, which makes the element particularly suitable for modelling laminated composite components while preserving solid-element connectivity at the interfaces with the adjacent isotropic regions discretised by SOLID186.

2.2.1 Modal Analysis

Modal analysis characterises the undamped free-vibration response of a structure, providing the natural frequencies and the associated mode shapes which constitute an orthogonal basis for its linear dynamic behaviour. A detailed treatment of the topic can be found in [Clough & Penzien \(1995\)](#) and [Géradin & Rixen \(2015\)](#). Starting from the assembled equation of motion (2.11) in the absence of damping and external loading:

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{0}\}. \quad (2.12)$$

and postulating a harmonic solution of the form $\{\mathbf{u}(t)\} = \{\boldsymbol{\phi}\} e^{i\omega t}$, the problem reduces to the generalised eigenvalue problem

$$([\mathbf{K}] - \omega_i^2 [\mathbf{M}]) \{\boldsymbol{\phi}_i\} = \{\mathbf{0}\}, \quad (2.13)$$

whose non-trivial solutions $(\omega_i^2, \{\boldsymbol{\phi}_i\})$ are, respectively, the squared natural angular frequencies and the corresponding eigenvectors (mode shapes) of the structure. The eigenvectors are conventionally mass-normalized, so that

$$\{\boldsymbol{\phi}_i\}^T [\mathbf{M}] \{\boldsymbol{\phi}_j\} = \delta_{ij}, \quad \{\boldsymbol{\phi}_i\}^T [\mathbf{K}] \{\boldsymbol{\phi}_j\} = \omega_i^2 \delta_{ij}, \quad (2.14)$$

with δ_{ij} the Kronecker symbol.

The eigenproblem of Equation 2.13 is solved in ANSYS® Mechanical by means of the Block Lanczos algorithm, developed by [Lanczos \(1950\)](#) and [Grimes, Lewis & Simon \(1994\)](#).

2.2.2 Harmonic Analysis

Harmonic response analyses are used to determine the steady-state response of a linear structure to loads that vary sinusoidally (harmonically) with time ([ANSYS, INC., 2023b](#)). For a harmonic load $\{\mathbf{F}(t)\} = \{\mathbf{F}_0\} e^{i\omega t}$ at angular frequency ω , the steady-state ansatz $\{\mathbf{u}(t)\} = \{\mathbf{U}\} e^{i\omega t}$ transforms Equation 2.11 into the complex algebraic system:

$$(-\omega^2 [\mathbf{M}] + i\omega [\mathbf{C}] + [\mathbf{K}]) \{\mathbf{U}\} = \{\mathbf{F}_0\}. \quad (2.15)$$

Direct solution of Equation 2.15 is called the full method, it is robust but becomes computationally expensive when the response must be evaluated at a large number of excitation frequencies. The *mode-superposition* approach exploits the modal basis previously obtained from the eigenanalysis to project the equations of motion onto a reduced set of $m \ll n$ retained modes, in which n is the total number of degrees of freedom of the model. The physical response is expressed as a linear combination of mode shapes,

$$\{\mathbf{u}(t)\} = \sum_{i=1}^m \{\phi_i\} y_i(t) = [\Phi] \{\mathbf{y}(t)\}, \quad (2.16)$$

where $[\Phi] = [\{\phi_1\} \{\phi_2\} \cdots \{\phi_m\}]$ is the modal matrix and $\{\mathbf{y}(t)\}$ is the vector of modal coordinates. Substitution into Equation 2.11, premultiplication by $[\Phi]^T$ and exploitation of the orthogonality of the mode shapes with respect to $[\mathbf{M}]$ and $[\mathbf{K}]$ decouple the coupled system into a set of single-degree-of-freedom modal equations,

$$\ddot{y}_i(t) + 2\zeta_i \omega_i \dot{y}_i(t) + \omega_i^2 y_i(t) = f_i(t), \quad i = 1, \dots, m, \quad (2.17)$$

where $f_i(t) = \{\phi_i\}^T \{\mathbf{F}(t)\}$ is the modal force and ζ_i is the modal damping ratio associated with the i -th mode. For harmonic excitation, the closed-form solution of Equation 2.17 yields the complex modal amplitude

$$y_i(\omega) = \frac{f_{i,0}}{\omega_i^2 - \omega^2 + 2i \zeta_i \omega_i \omega}, \quad (2.18)$$

and the physical response is recovered by back-transformation, $\{\mathbf{U}(\Omega)\} = [\Phi] \{\mathbf{y}(\Omega)\}$. The cost of evaluating the response across a frequency sweep is therefore dominated by the (one-off) eigensolution rather than by the number of frequency points, which makes mode superposition particularly efficient for frequency-response problems with many excitation frequencies and a limited bandwidth of interest.

Since experimentally resolved, element-wise damping data are rarely available, damping is introduced in this work through a constant modal damping ratio applied uniformly to all retained modes by means of the `DMPRAT` command of [ANSYS® Parametric Design Language \(APDL\)](#), as described in [ANSYS, Inc. \(2023b\)](#). The `DMPRAT` command specifies a single value of damping ζ that is assigned to every mode in the superposition, so that $\zeta_i = \zeta$ for all i . This frequency-independent, structurally uniform damping model is a reasonable first approximation for lightly damped metallic structures in the absence of detailed information about the underlying loss mechanisms, and guarantees that all retained modes contribute to the frequency-response function with finite peak amplitudes.

2.3 Genetic algorithm optimization

An optimization problem consists in finding the values of a set of design variables $\mathbf{x}$ that minimize a scalar objective function $f(\mathbf{x})$ subject to equality, inequality and bound constraints. This problem is mathematically written as:

$$\begin{aligned} \min_{\mathbf{x} \in \mathbb{R}^n} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \dots, m \\ & h_j(\mathbf{x}) = 0, \quad j = 1, 2, \dots, p \\ & x_k^L \leq x_k \leq x_k^U, \quad k = 1, 2, \dots, n \end{aligned} \quad (2.19)$$

where $g_i(\mathbf{x})$ and $h_j(\mathbf{x})$ denote, respectively, the m inequality and p equality constraint functions, and x_k^L and x_k^U are the lower and upper bounds on the k -th design variable. The set of all $\mathbf{x}$ satisfying these conditions defines the feasible region

$$\Omega = \left\{ \mathbf{x} \in \mathbb{R}^n \mid g_i(\mathbf{x}) \leq 0, h_j(\mathbf{x}) = 0, x_k^L \leq x_k \leq x_k^U \right\}. \quad (2.20)$$

[Nocedal & Wright \(2006\)](#) and [Rao \(2019\)](#) are classical references in engineering optimization. Gradient-based methods are computationally efficient when f is smooth and differentiable, but they are highly sensitive to the choice of the initial point, typically converge to local minima, and require derivative information that may be unavailable or unreliable when f is evaluated through a black-box numerical model.

[GA](#) are stochastic, population-based metaheuristics inspired by the principles of natural selection and biological evolution. They were originally introduced by [Holland \(1975\)](#) and later popularized for engineering optimization problems by [Goldberg \(1989\)](#). A [GA](#) operates on a population of candidate solutions (individuals), each encoded as a vector of design variables, and iteratively evolves it through three operators that mimic biological processes: *selection* of the fittest individuals according to the objective function, *crossover* that recombines pairs of parents to produce new offspring, and *mutation* that introduces random perturbations to explore different solutions. Because [GAs](#) require only objective function evaluations and no derivative information, they are well suited to non-smooth, non-convex and discontinuous problems for which gradient-based methods are inapplicable, at the cost of a generally higher number of function evaluations.

In this work the optimization is carried out with the `ga` function of the MATLAB® Global Optimization Toolbox ([MATHWORKS INC., 2023](#)). This implementation also supports parallel computing, allowing the objective-function evaluations of each generation to be distributed across multiple workers, which significantly reduced the overall computational time required by the optimization process. The objective functions were designed to tune the antiresonances produced by the candidate [MMDVAs](#) to the desired frequencies.

The specific implementation used in this work follows the algorithm described in the MATLAB® Global Optimization Toolbox documentation ([MATHWORKS INC., 2026](#)). At each generation t , the **ga** solver operates on a population (P) of N_p individuals

$$P^{(t)} = \{\mathbf{x}_1^{(t)}, \dots, \mathbf{x}_{N_p}^{(t)}\}, \quad \mathbf{x}_l^{(t)} \in \mathbb{R}^n, \quad (2.21)$$

encoded directly as real-valued design vectors, and produces the next generation $P^{(t+1)}$ through the following steps.

Fitness scaling: The raw objective-function values $f(\mathbf{x}_l^{(t)})$ are first converted into scaled *expectation values* $E_l^{(t)}$, which determine each individual's likelihood of being selected as a parent. By default, **ga** uses rank-based scaling, in which the expectation of an individual depends only on its rank r_l (position after sorting by fitness) within the population rather than on the raw value of f itself,

$$E_l^{(t)} = \frac{1}{\sqrt{r_l}}, \quad r_l \in \{1, 2, \dots, N_p\}, \quad (2.22)$$

normalized so that $\sum_l E_l^{(t)} = N_p$. Because it depends on rank rather than the magnitude of f , this scaling prevents a small number of individuals with disproportionately good fitness values from dominating the selection process.

Selection: Parents are drawn from the scaled population using the default *stochastic uniform* procedure. A line is laid out in which each individual l occupies a segment of length proportional to $E_l^{(t)}$, with cumulative segment boundaries ($S_l^{(t)}$),

$$S_l^{(t)} = \sum_{k=1}^l E_k^{(t)}, \quad l = 1, \dots, N_p. \quad (2.23)$$

The total line length is $S_{N_p}^{(t)}$, which is divided into N_p equal steps of size

$$\Delta = \frac{S_{N_p}^{(t)}}{N_p}. \quad (2.24)$$

Starting from a random offset $u_0 \sim \mathcal{U}(0, \Delta)$, the algorithm places N_p evenly spaced points along the line,

$$u_j = u_0 + (j - 1)\Delta, \quad j = 1, \dots, N_p, \quad (2.25)$$

and selects, for each j , the individual whose segment contains u_j . Because segment length is proportional to $E_l^{(t)}$, fitter individuals occupy longer segments and are therefore more likely to be selected, and may be selected more than once.

Elitism: The N_e individuals with the best fitness in $P^{(t)}$ (set by the **EliteCount** option) are copied unchanged into $P^{(t+1)}$,

$$P_{\text{elite}}^{(t+1)} = \{\mathbf{x}_{(1)}^{(t)}, \dots, \mathbf{x}_{(N_e)}^{(t)}\}, \quad (2.26)$$

where $\mathbf{x}_{(1)}^{(t)}, \dots, \mathbf{x}_{(N_e)}^{(t)}$ denote the N_e best-ranked individuals of $P^{(t)}$. This guarantees that the best fitness value is non-increasing across generations.

Crossover children: A fraction p_c (the `CrossoverFraction` option) of the remaining $N_p - N_e$ individuals are generated by crossover. For unconstrained or bound-constrained problems, the default *scattered* crossover independently selects, for each coordinate i , the gene of one of the two parents $\mathbf{x}_a^{(t)}, \mathbf{x}_b^{(t)}$ at random,

$$x_{\text{off},i} = \begin{cases} x_{a,i}^{(t)} & \text{with probability } 0.5 \\ x_{b,i}^{(t)} & \text{with probability } 0.5 \end{cases}, \quad i = 1, \dots, n \quad (2.27)$$

Mutation children: The remaining $N_p - N_e - N_{\text{cross}}$ individuals are produced by mutation, applied to a single parent. In this work, the Gaussian mutation function was used, which adds a random perturbation drawn from a Gaussian distribution with zero mean to each entry of the parent vector:

$$x_{\text{off},i} = x_{a,i}^{(t)} + z_i, \quad z_i \sim \mathcal{N}(0, \sigma_{i,t}^2), \quad i = 1, \dots, n \quad (2.28)$$

The standard deviation $\sigma_{i,t}$ decreases across generations according to the recursive rule

$$\sigma_{i,t} = \sigma_{i,t-1} \left(1 - \text{Shrink} \cdot \frac{t}{t_{\text{max}}} \right) \quad (2.29)$$

where t_{max} is the maximum number of generations (`MaxGenerations`) and `Shrink` is a user-defined parameter (default value 1) controlling the rate at which the standard deviation decreases; a value of 1 causes $\sigma_{i,t}$ to decrease linearly to zero by the final generation. The initial standard deviation $\sigma_{i,0}$ is set by the `Scale` parameter (default value 1) as $\sigma_{i,0} = \text{Scale} \cdot (v_{i,2} - v_{i,1})$, where $v_{i,1}$ and $v_{i,2}$ are the lower and upper bounds of the initial population range for coordinate i (`InitialPopulationRange`). This progressive reduction in mutation step size allows broader exploration of the design space in early generations and finer local refinement as the population converges in later generations.

Generational update: The new population is finally assembled as

$$P^{(t+1)} = P_{\text{elite}}^{(t+1)} \cup P_{\text{crossover}}^{(t+1)} \cup P_{\text{mutation}}^{(t+1)}, \quad |P^{(t+1)}| = N_p \quad (2.30)$$

with $|P_{\text{crossover}}^{(t+1)}| = \text{round}[p_c(N_p - N_e)]$ and the remainder assigned to mutation. For example, with $N_p = 20$, $N_e = 2$, and $p_c = 0.8$, the toolbox produces 2 elite, 14 crossover ($0.8 \times 18 = 14.4$, rounded down), and 4 mutation children.

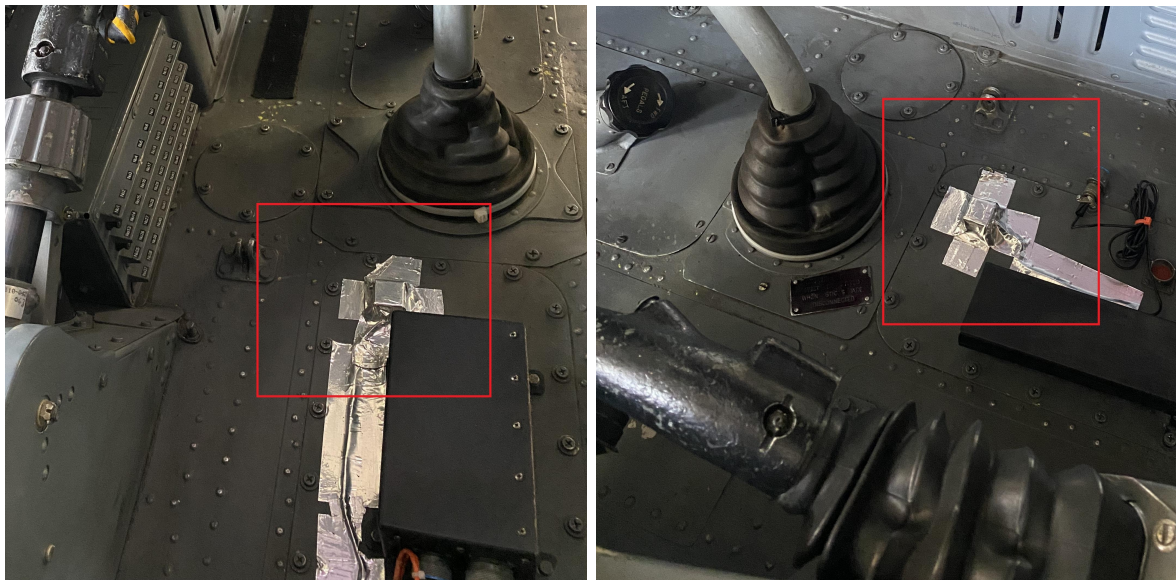
Stopping criteria: The algorithm terminates when any one of several conditions is satisfied: a maximum number of generations is reached, the best fitness value falls below a prescribed limit, or the relative change in the best fitness value averaged over a fixed window of generations falls below a function tolerance.

3 EXPERIMENTAL VIBRATION CHARACTERIZATION

The data acquisition occurred during a flight test campaign conducted at the [Brazilian Flight Test and Research Institute \(IPEV\)](#) with the Mk21B Wild Lynx. The campaign's primary goal was to evaluate aircraft performance in specific operational procedures. As an additional evaluation, accelerometers were installed to collect data for this study.

3.1 Flight test data acquisition

Two 3-axis accelerometers were placed on the cabin floor. Accelerometer 1 was placed on the floor under the pilot's seat (Figure 3.1a) and accelerometer 2 was placed under co-pilot's seat (Figure 3.1b). Both accelerometers are CXL10GP3-R modules. The signals were recorded at 960 Hz sampling rate. The sensors were oriented such that the Y-axis points forward (longitudinal), the X-axis points to the left (lateral), and the Z-axis points downward (vertical).



(a) Location of accelerometer 1.

(b) Location of accelerometer 2.

Figure 3.1 – Accelerometer positioning in the Lynx helicopter.

Figure 3.2 shows the position of the cockpit floor, where the accelerometers were installed.

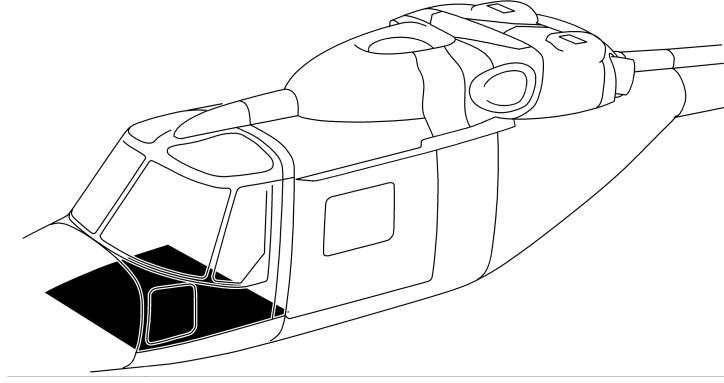


Figure 3.2 – Location of the plate where the accelerometers were positioned.

Source: Adapted from [Leonardo Helicopters \(2023\)](#)

3.2 Signal processing

According to [Scheffer \(2004\)](#), in the range from 10 Hz to 1000 Hz, velocity gives a good indication of the severity of vibration. The military standard ADS-27A-SP ([US Army Aviation and Missile Command, 1996](#)) shows specifications in terms of velocity in [inches per second \(IPS\)](#), as it is common practice in helicopter industry. For that reason, vibration levels were converted to [IPS](#). The integration from acceleration to velocity was done in the frequency domain as in Equation 3.1.

$$V(\omega) = \frac{A(\omega)}{i\omega} \quad (3.1)$$

To minimize spectral leakage ([HARRIS; PIERSOL, 2002](#)), the raw time-domain signals were tapered using a Hanning window ([SCHEFFER, 2004](#)) before the computation of the [fast Fourier transform \(FFT\)](#). Prior to integration, the time-domain acceleration signals were high-pass filtered to suppress DC offset and low-frequency drift, which would otherwise be amplified by the $1/i\omega$ factor in the frequency-domain integration. A 5th order Butterworth filter with a cutoff frequency of 2 Hz was designed using MATLAB®'s `butter` function and applied via `filtfilt` function to achieve zero-phase response.

Following the described [FFT](#) and integration methodology, the dominant spectral components were first identified by computing a single [FFT](#) over the entire time window of interest, after Hanning tapering and high-pass filtering as described above. The resulting velocity spectrum was inspected to locate the peaks corresponding to the vibration components of interest.

To track how these components evolve across flight conditions, a combined time–frequency representation was then adopted. The time-domain signals were segmented into 4096-sample windows with 50 % overlap, and each segment was tapered with a Hanning window before computing the [FFT](#). For every segment, the magnitude spectrum was scanned with MATLAB®'s `findpeaks` function to identify the local maxima corresponding

to the previously selected components. The [root mean square \(RMS\)](#) amplitude of each peak was then plotted as a function of time across the selected flight condition windows, yielding a time-resolved view of the vibration content at each frequency of interest. The selected flight conditions for the analysis were:

- 50 feet hover;
- 80 knots leveled flight;
- 100 knots leveled flight.

3.3 Discussion of test results

A time-domain plot of the accelerometer data is displayed in Figure 3.3. [Welsh \(2018\)](#) reports that vibration levels in conventional helicopters are typically between 0.1 g and 0.2 g at most cockpit and cabin locations. The recorded signals reach values above 1 g for accelerometer 1, which consistently exhibited the higher amplitudes of the two sensors. This discrepancy between the two accelerometers is likely attributable to differences in the local modal response and proximity to the dominant excitation sources at each mounting location. In any case, the measured levels exceed the reference values reported in the literature, indicating significant potential for vibration mitigation.

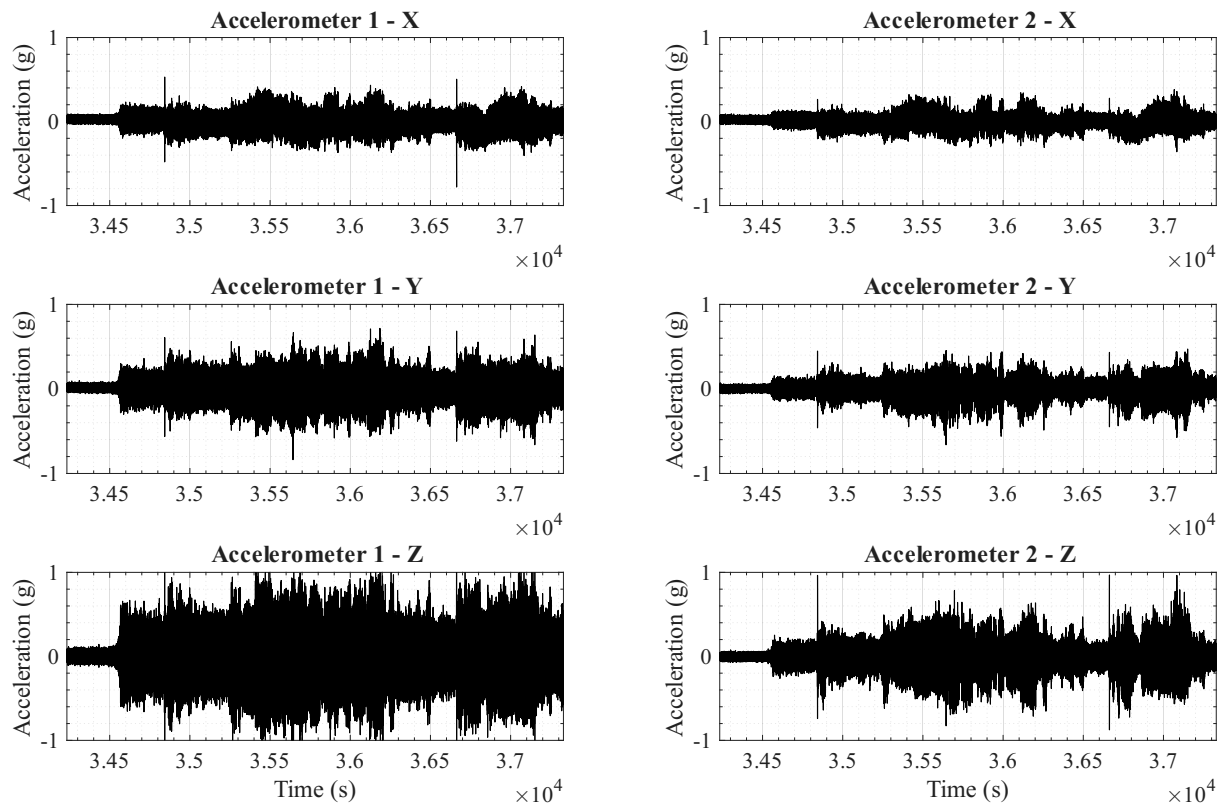


Figure 3.3 – Time-domain accelerometer signals over the complete flight duration.

The Z-axis of accelerometer 1 exhibited the highest vibration amplitudes among the measured channels and was therefore selected for the full-window spectral analysis. The resulting acceleration and velocity spectra are shown in Figures 3.4 and 3.5. This initial analysis identified the frequencies at which the most critical amplitude peaks occur and their excitation sources, determined through correlation with rotor harmonic frequencies. These results guided the selection of frequencies for a more detailed analysis. As described by Scheffer (2004), within the 10 Hz to 1000 Hz band, vibration severity is better expressed in the velocity spectrum, which was therefore used to select the target frequencies. Therefore, the main sources of excitation are:

- 5.5 Hz: main rotor 1/rev (fundamental);
- 11 Hz: main rotor 2/rev;
- 22 Hz: main rotor 4/rev;
- 33 Hz: main rotor 6/rev;
- 44 Hz: main rotor 8/rev;
- 66 Hz: main rotor 12/rev;
- 111 Hz: tail rotor 4/rev.

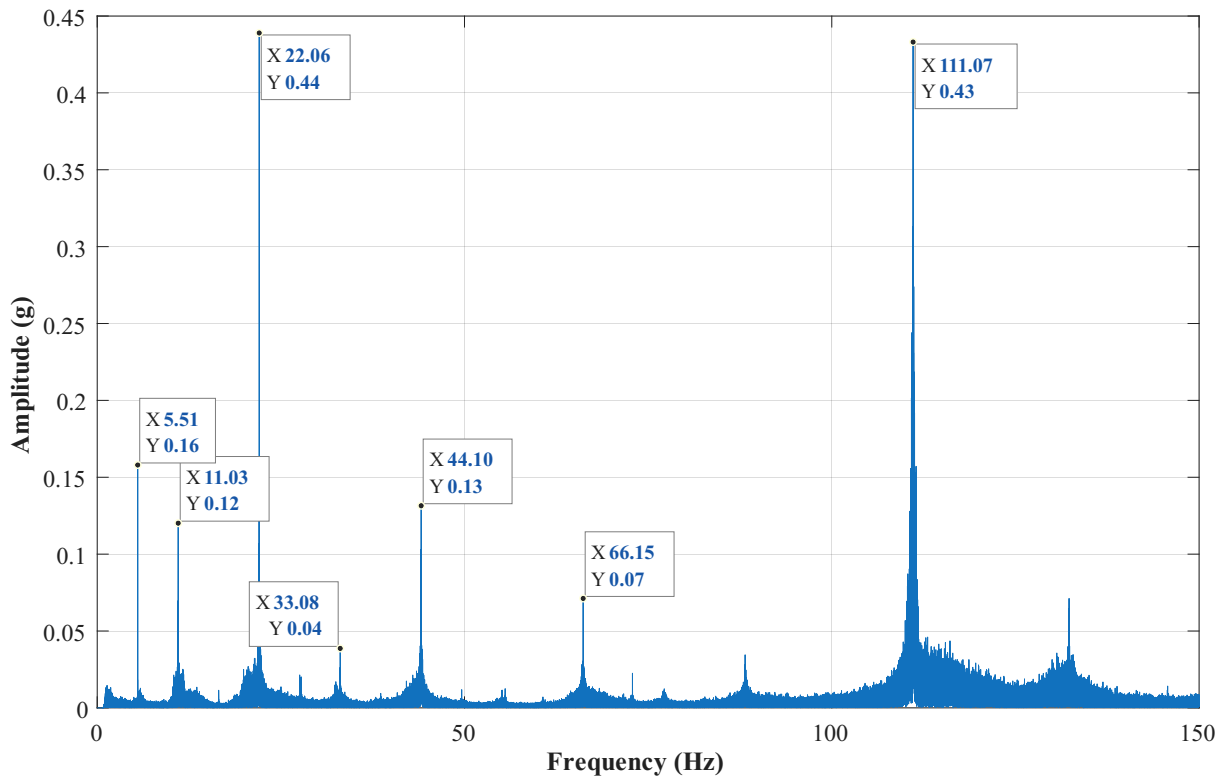


Figure 3.4 – Amplitude spectrum of acceleration for Z axis of accelerometer 1.

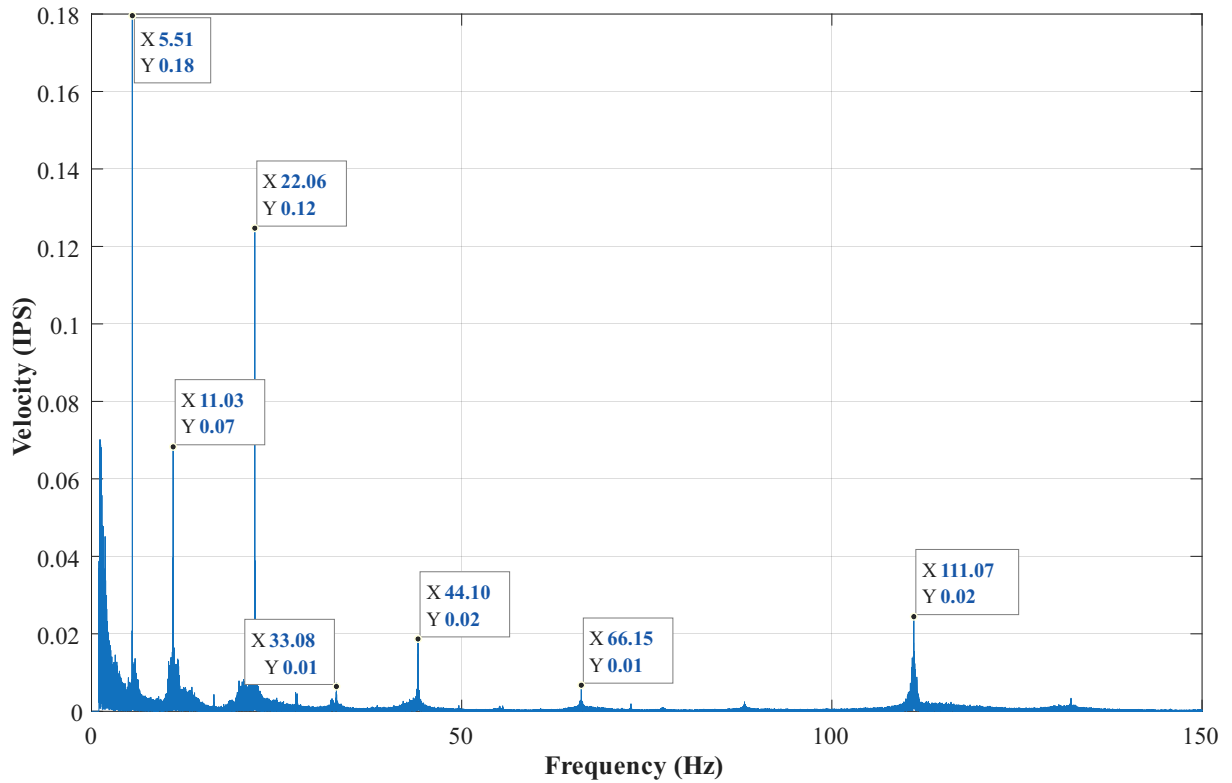


Figure 3.5 – Amplitude spectrum of velocity for Z axis of accelerometer 1.

The 5.5 Hz frequency in the velocity spectrum is artificially amplified by the $1/i\omega$ factor introduced by integration, and lies outside the 10 Hz to 1000 Hz band. Moreover, Chapter 5 demonstrates that achieving low natural frequencies may prove challenging due to the combination of low stiffness and high mass.

The combined time–frequency analysis shown in Figures 3.6, 3.7, and 3.8 reveals that the tail rotor 4/rev amplitude was consistently higher at accelerometer 1 than at accelerometer 2. The amplitude at this frequency also varied with flight condition in both sensors, reaching its maximum in hover. This is expected, since the tail rotor must counteract the full main rotor torque in hover, with no offloading from the vertical fin sideforce that develops in forward flight as discussed in Padfield (2008).

All selected main rotor frequencies exhibit significant amplitudes across the flight conditions analyzed, particularly the 4/rev main rotor component, which remains prominent despite the attenuation provided by the flexispring absorber (Figure 1.6). $N_b = 4$ for both main rotor and tail rotor, so these are in fact expected as high amplitude harmonics according to literature.

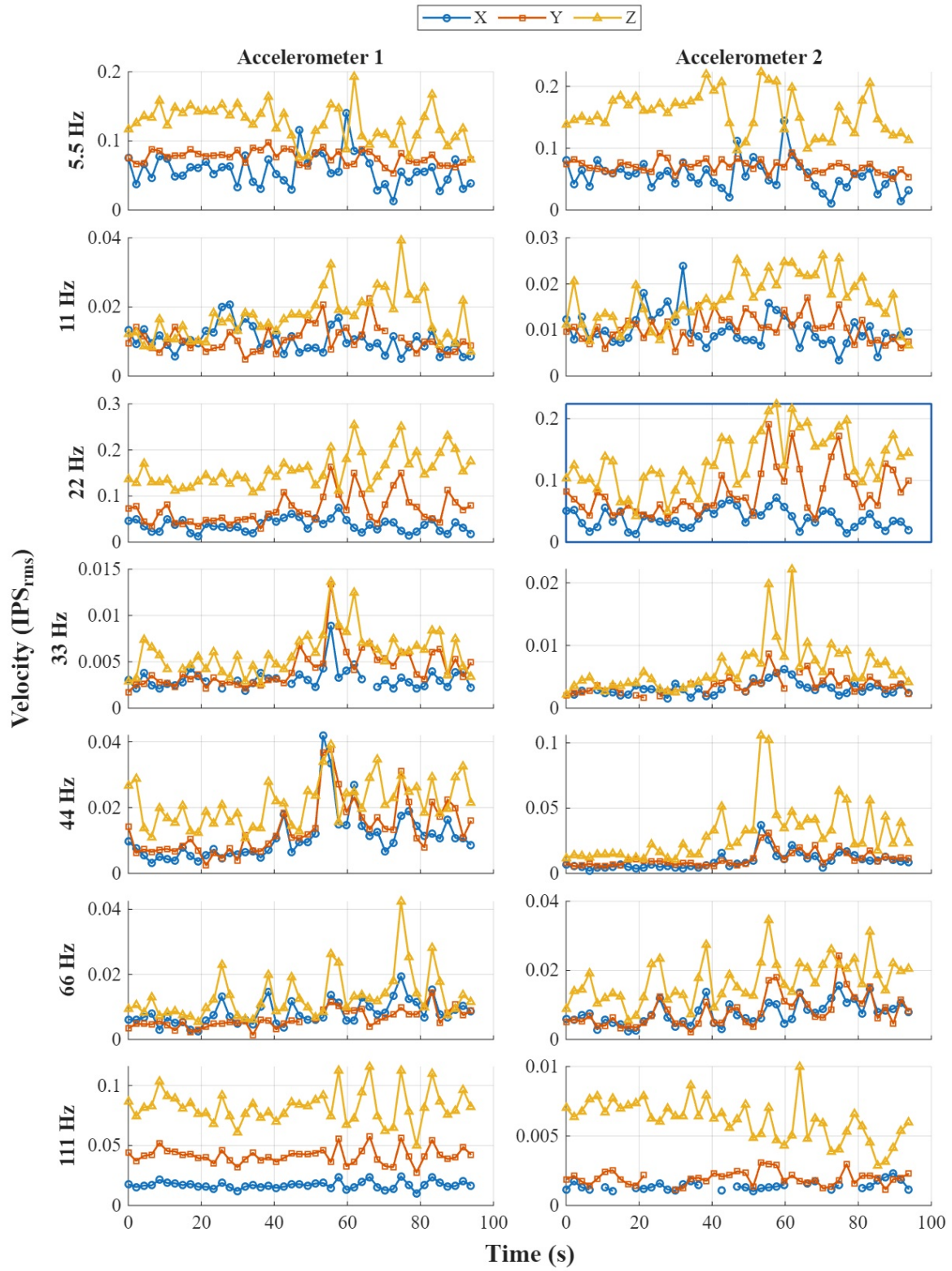


Figure 3.6 – Hover data analysis.

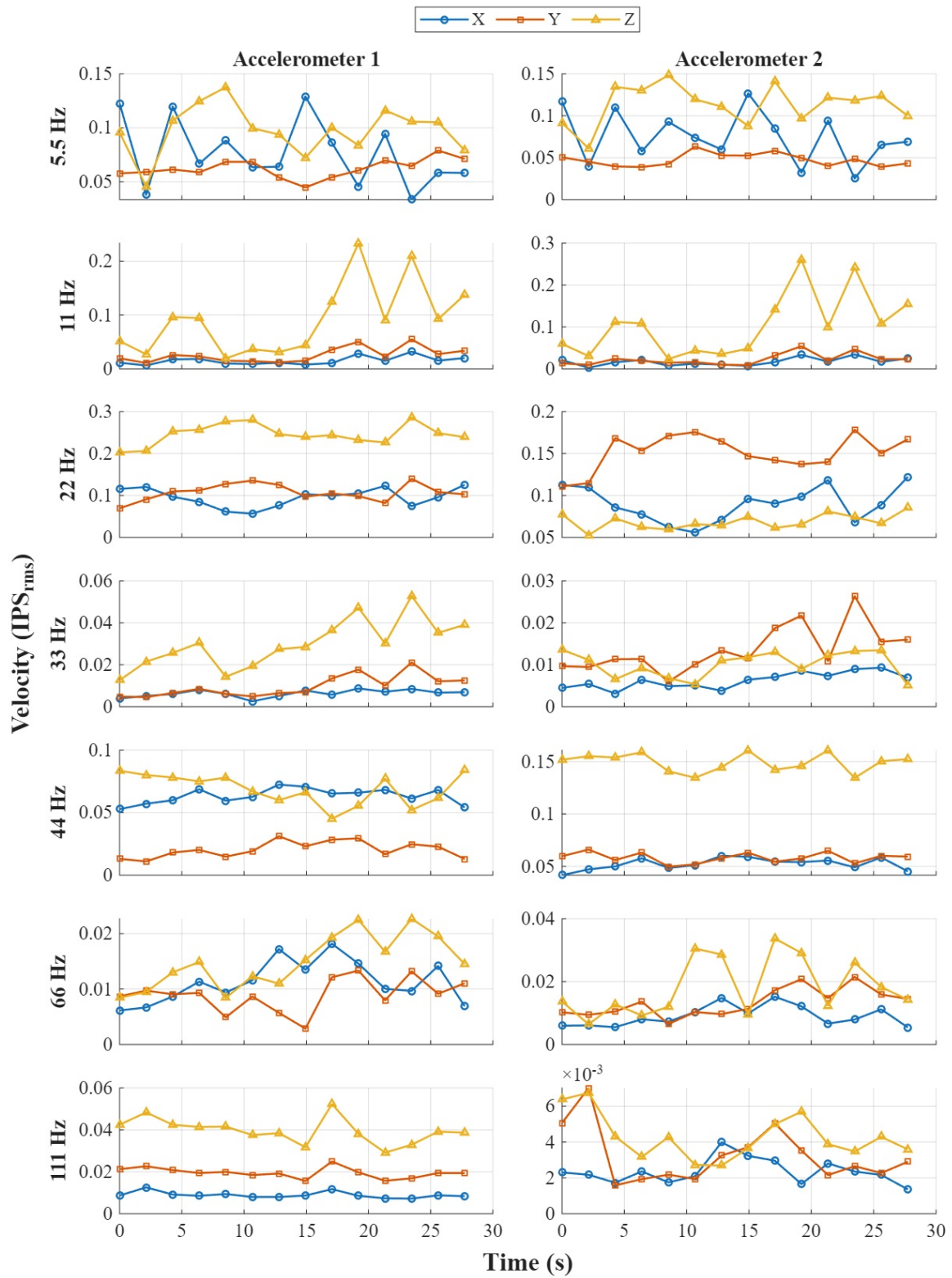


Figure 3.7 – 80 knots leveled flight data analysis.

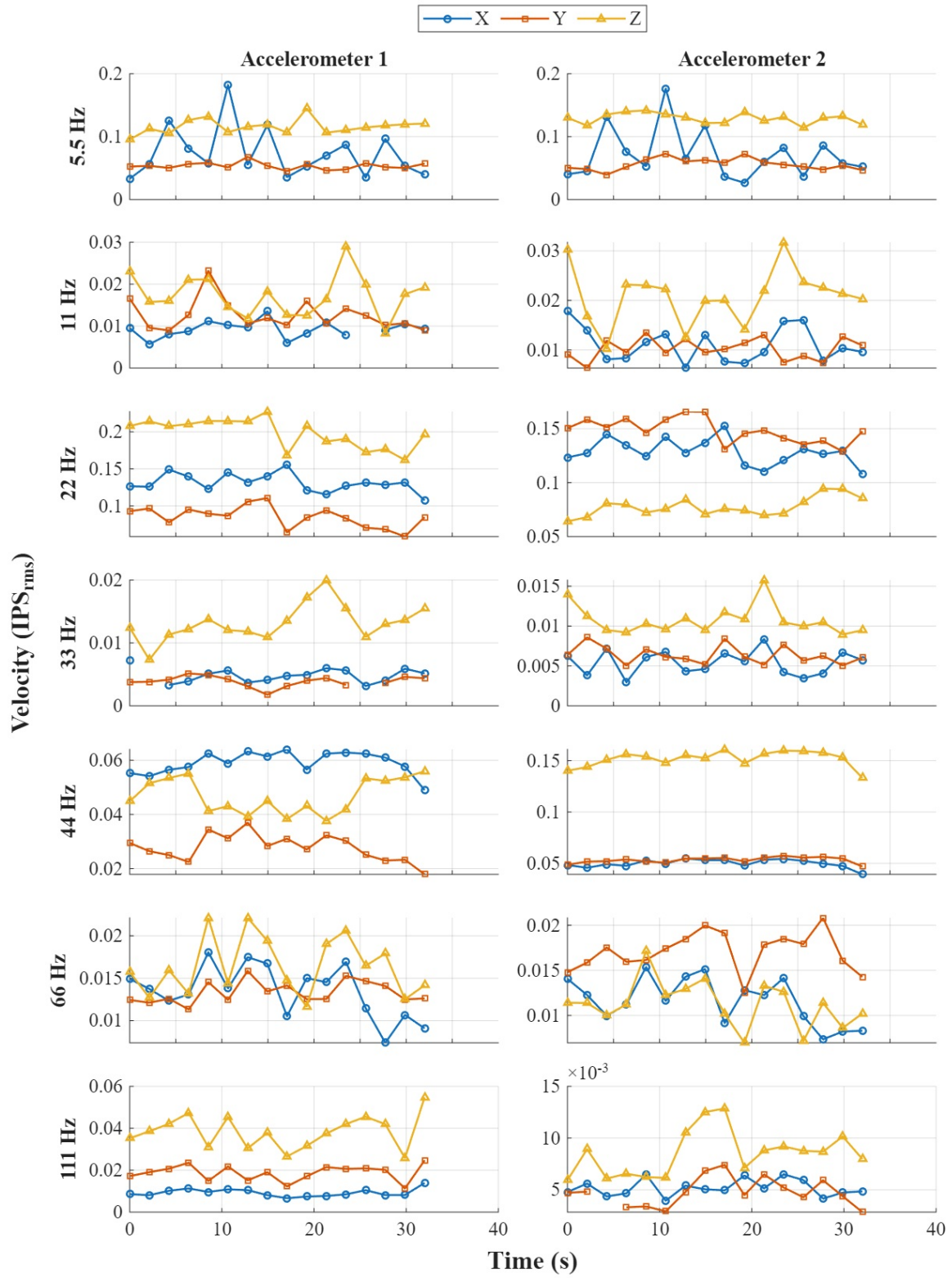


Figure 3.8 – 100 knots leveled flight data analysis.

4 DESIGN METHODOLOGY

The design methodology proposed in this work combines an experimental characterization described in Chapter 3 with a numerical design loop based on FEM and GA optimization. An overview of the complete workflow is presented in Figure 4.1.

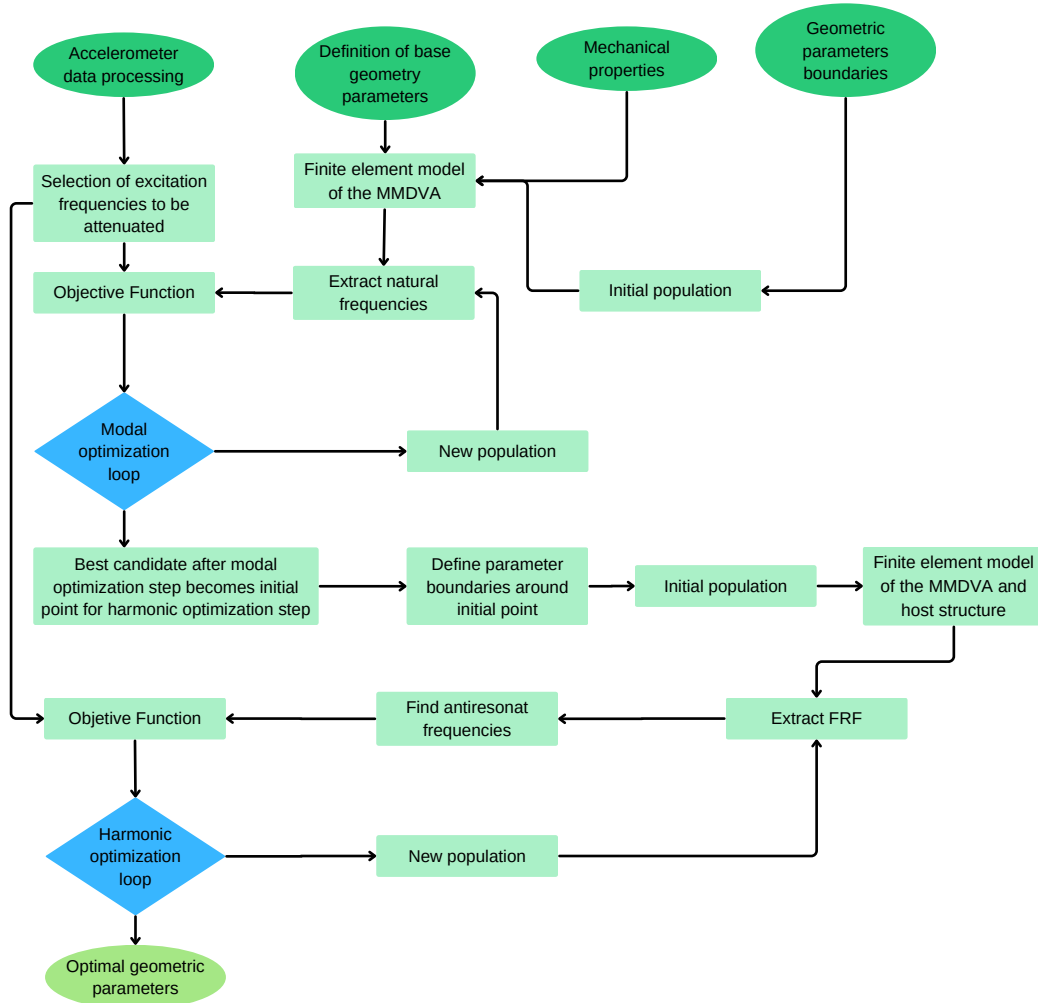


Figure 4.1 – Flowchart.

4.1 Design philosophy and geometric configuration

Two candidate geometries were proposed, using carbon fiber leaf springs integrated with steel inertial masses. Scheffer (2004, Section 6.4.1) presents a theoretical analysis of cantilever structures acting as vibration absorbers, and it served as inspiration for both base geometries in the use of cantilever structures. Figure 1.2 also shows a cantilever-based vibration absorber implemented in a Black Hawk helicopter. The combination of

beams to form a pring was inspired by the design used the Sea King absorber shown in Figure 1.1, a similar idea was implemented in Geometry 2. Based on these ideas, detailed drawings and parameter distributions were proposed by the author.

White (1979) discussed the importance of selecting materials with high stiffness and low density for spring components; for this reason, carbon fiber was selected, as it offers a considerably higher specific stiffness (stiffness-to-density ratio) than conventional metallic alternatives such as steel. Moreover, these materials are already well established in the aircraft industry, where their high specific stiffness and strength have led them to progressively displace conventional metallic alloys in structural applications, many examples are described by Soutis (2005). Specific application of carbon fiber as springs is seen in Karad, Sonawwanay & Bachhav (2023) and Soliman (2021).

A complementary strategy to mitigate the mass penalty inherent to dynamic vibration absorbers consists of exploiting more than one vibration mode of the same elastic structure. A cantilever leaf spring carrying inertial masses possesses several distinct natural modes, bending and torsion being the most readily exploited, each with its own natural frequency, governed by the stiffness associated with that particular deformation pattern and by the corresponding modal mass. By tuning the bending mode to one target frequency and the torsional mode to another, a single device is able to attenuate vibration at multiple frequencies simultaneously, which is especially advantageous when the excitation comprises several discrete harmonics, as occurs with the rotor-induced loads in helicopters. In this manner, the same installed mass contributes effective inertia to more than one mode, rather than performing useful work at a single frequency as in a classical single-mode absorber. The same principle may be pursued by distributing the inertial masses along the spring so that each target mode mobilizes a different subset of them; since the effective modal mass of a given mode corresponds to only a fraction of the total physical mass, tailoring the mode shapes such that distinct masses participate predominantly in distinct modes allows an adequate effective mass to be obtained for each targeted frequency without dedicating an independent absorber to each one. Both approaches improve the ratio between the useful effective mass, summed over the targeted frequencies, and the total mass added to the system, thereby optimizing the mass efficiency of the absorber.

4.1.1 Base Geometry 1

This proposed design consists of a carbon fiber cantilever beam with a steel block acting as the tip mass (Figure 4.2). The composite layup was organized into sets of two plies, each featuring a specific fiber orientation and a nominal thickness of 0.2 mm, resulting in 0.4 mm per set. The optimization objective was to tune the flexural and torsional natural frequencies of the MMDVA so that the resulting antiresonances of the coupled system coincide with the target excitation frequencies. Each unit of Geometry 1

provides two antiresonances, one from its flexural mode and one from its torsional mode. Two units were therefore combined to target the four required frequencies, connected by a square hollow section.

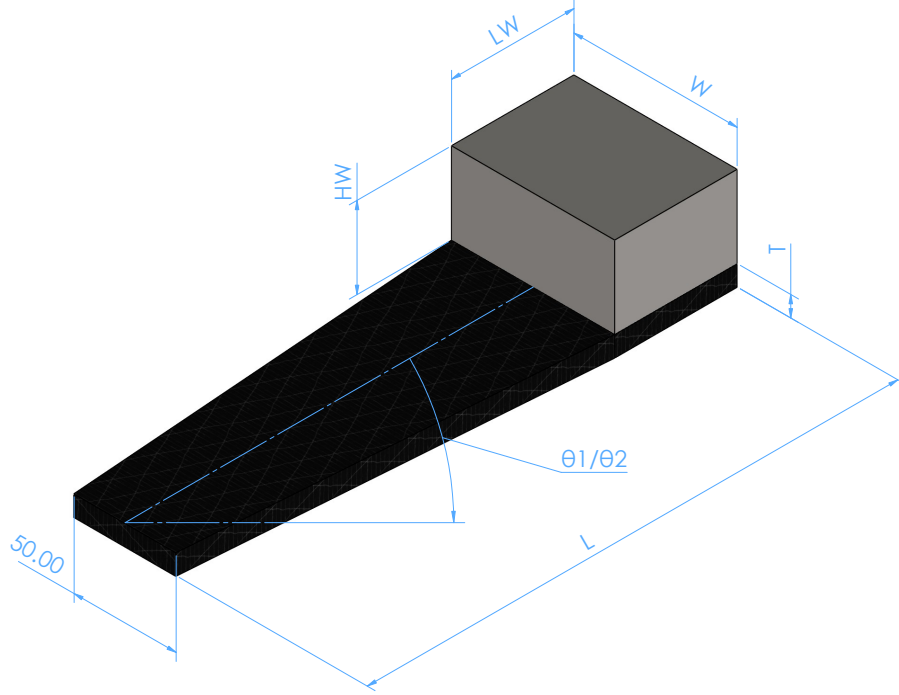


Figure 4.2 – Base Geometry 1 parameters.

Table 4.1 – Design parameters and optimization bounds for Base Geometry 1.

Parameter	Description	Unit	Min	Figure 4.2	Max
N^1	Number of composite layer sets	-	5	25	25
θ_1	Fiber orientation 1	$^\circ$	0	-	179
θ_2	Fiber orientation 2	$^\circ$	0	-	179
L	Effective length	mm	100	260	300
LW	Length of the weight block	mm	10	60	60
HW	Height of the weight block	mm	10	40	60
W	Tip width	mm	10	80	80

¹ Beam thickness (T) is calculated according to the number of layers.

4.1.2 Base Geometry 2

This configuration is illustrated in Figure 4.3 and functions as a distributed multi-degree-of-freedom mass-spring system. The central hub acts as a primary inertial element, while the two orthogonal arms with tip masses act as coupled sub-absorbers. This design leverages the added weight penalty effectively by distributing inertia to simultaneously act in multiple flexural or torsional motion modes. All composite components are organized

in sets of two 0.2 mm plies resulting in 0.4 mm nominal thickness per set, consistent with Base Geometry 1. The four beams that support the central weight share the same layup configuration, while each cantilever arm has its unique orientation set up.

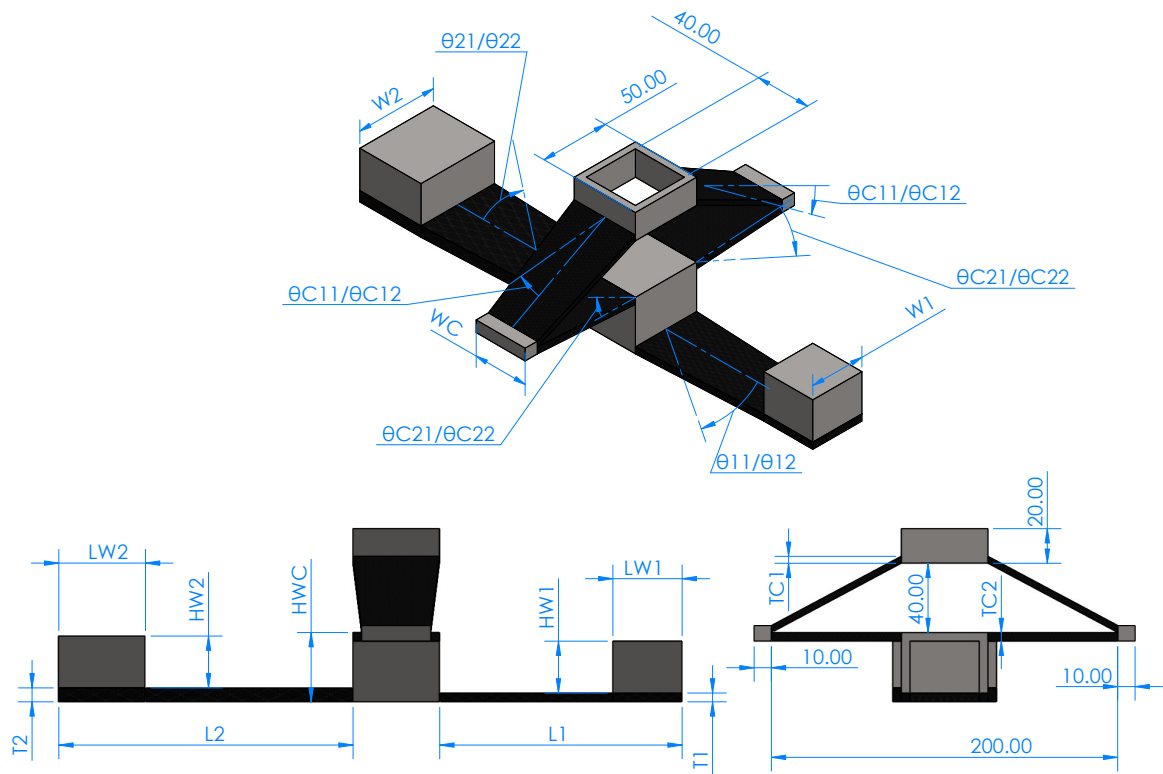


Figure 4.3 – Base Geometry 2 parameters.

Table 4.2 – Design parameters and optimization bounds for Base Geometry 2.

Parameter	Description	Unit	Min	Fig. 4.3	Max
$NC1$ ¹	Number of layer sets in thickness TC1	-	5	10	25
$\theta C11$	Fiber orientation 1 for TC1 thickness	°	0	-	179
$\theta C12$	Fiber orientation 2 for TC1 thickness	°	0	-	179
$NC2$ ¹	Number of layer sets in thickness TC2	-	5	12	25
$\theta C21$	Fiber orientation 1 for TC2 thickness	°	0	-	179
$\theta C22$	Fiber orientation 2 for TC2 thickness	°	0	-	179
WC	Central beams tip width	mm	25	40	75
HWC	Height of the central weight block	mm	10	40	60
$N1$ ¹	Number of layer sets in thickness T1	-	5	12	25
$\theta 11$	Fiber orientation 1 for T1 thickness	°	0	-	179
$\theta 12$	Fiber orientation 2 for T1 thickness	°	0	-	179
$L1$	Effective length (side 1)	mm	100	140	250
$LW1$	Length of the weight block (side 1)	mm	10	40	60
$HW1$	Height of the weight block (side 1)	mm	10	30	60
$W1$	Tip width (side 1)	mm	10	40	80
$N2$ ¹	Number of layer sets in thickness T2	-	5	20	25
$\theta 21$	Fiber orientation 1 for T2 thickness	°	0	-	179
$\theta 22$	Fiber orientation 2 for T2 thickness	°	0	-	179
$L2$	Effective length (side 2)	mm	100	170	250
$LW2$	Length of the weight block (side 2)	mm	10	50	60
$HW2$	Height of the weight block (side 2)	mm	10	30	60
$W2$	Tip width (side 2)	mm	10	60	80

¹ Beams thickness (TC1, TC2, T1 and T2) are calculated according to the corresponding number of layers.

4.2 Two-stage optimization strategy

The initial design stage focused on the modal characterization of the [MMDVA](#) in isolation. Numerical simulations were performed in [APDL](#) to extract the natural frequencies and mode shapes of the candidate geometries. By utilizing a Block Lanczos eigensolver, the geometric parameters were iteratively adjusted in MATLAB® to align the first flexural and first torsional natural frequencies with the target excitation harmonics.

However, the natural frequencies obtained from the isolated modal analysis serve only as primary tuning targets. When the absorber is integrated into the host structure, the resulting antiresonant peaks do not coincide exactly with these isolated frequencies. This discrepancy arises from the dynamic coupling between the systems and the influence of structural damping, which shifts the points of minimum transmissibility.

To account for these effects, a secondary optimization stage was implemented using harmonic analysis based on the modal superposition method. In this phase, the [MMDVAs](#) were numerically attached to a representative host beam within the [APDL](#) environment.

The host beam was modeled with clamped-clamped boundary conditions at both longitudinal extremities to represent a typical constrained airframe segment. The harmonic response was evaluated over the operational frequency range to observe the actual formation of the antiresonant notches. Specifically, a harmonic excitation force was applied to a single mid-span node, and the displacement FRF at that same point was evaluated. The dimensions of the host beam were selected such that its natural frequencies are well-separated from the excitation frequencies. Specifically, the fundamental mode of the host beam was set at 485.65 Hz, ensuring no interference occurs within the operational frequency range of interest (up to 111 Hz). This reflects standard helicopter structural design practices and ensures that the analysis focuses primarily on the influence of the absorber's antiresonances rather than host structure resonances. A damping ratio of 0.5 % was set for the harmonic analyses, this choice is further explained in Section 5.1

Specific design strategies were implemented for each geometry as follows:

- **Base Geometry 1:** The target excitation frequencies were identified as 11.03 Hz, 22.06 Hz, 44.1 Hz and 111.07 Hz, The 5.5 Hz harmonic was excluded as a target frequency because the low-frequency requirement would necessitate a beam with insufficient stiffness, potentially compromising the structural integrity and durability of the carbon fiber spring. The configuration shown in Fig. 4.2 was subjected to a two-stage optimization process: initially, the geometry was optimized for two separate pairs of target frequencies. Subsequently, these preliminary designs were integrated into a unified model and fine-tuned through harmonic analysis of the coupled system (absorber and host beam) across the entire operational frequency range. For this configuration, the flexural modes primarily generated antiresonances in the vertical (Z -axis) direction, whereas the torsional modes introduced antiresonances in the lateral (Y -axis) plane. A square hollow section was employed as the structural interface between the Geometry 1 components and the host beam.
- **Base Geometry 2:** The target excitation frequencies were changed to 11.03 Hz, 22.06 Hz, 44.1 Hz and 66.15 Hz in order to reduce computational optimization time by narrowing the frequency range for harmonic simulation. The configuration shown in Figure 4.3 was subjected to a two-stage optimization process consistent with Base Geometry 1. An initial analysis of the finite element model showed that the antiresonances are more evident along the vertical axis.

The optimal parameters were sought through a two-stage optimization routine using the GA in MATLAB®. In the first stage, a global search was executed to align the isolated natural frequencies ($f_{n,k}$) of the absorber with the target excitation harmonics ($f_{t,k}$). This modal-based optimization allowed for a rapid exploration of the geometric design space, identifying a single best candidate configuration for the dimensional parameters and the

composite layup orientation. The objective function and optimization settings are presented in Table 4.3 for Geometry 1 and in Table 4.4 for Geometry 2. The objective functions were designed to minimize the difference between the MMDVAs' natural frequencies and the targeted frequencies. The decision variables are the dimensions and fiber orientation angles described in Figures 4.2 and 4.3 subjected to the boundaries in Tables 4.1 and 4.2.

Table 4.3 – GA configuration and parameters for Base Geometry 1 modal optimization.

Parameter	Value
Population Size	50
Max Generations	30
Crossover Fraction	0.8
Integer Variables	N, θ_1, θ_2
Objective Function	$\sum_{k=1}^2 \left(\frac{f_{n,k} - f_{t,k}}{f_{t,k}} \right)^2$

Table 4.4 – GA configuration and parameters for Base Geometry 2 modal optimization.

Parameter	Value
Population Size	100
Max Generations	80
Crossover Fraction	0.8
Integer Variables	$NC1, \theta C11, \theta C12, NC2, \theta C21, \theta C22, N1, \theta 11, \theta 12, N2, \theta 21$ and $\theta 22$
Objective Function	$\sum_{k=1}^4 (f_{n,k} - f_{t,k})^2$

Following the identification of the optimal design vector obtained from the first-stage (modal) optimization (X_0), a second-stage refined harmonic optimization was implemented. During this phase, the MMDVAs were numerically coupled to the host beam model in APDL. This sequential approach ensured that the final geometry accounted for the frequency shifts induced by structural damping and system coupling, which are not captured by isolated modal analysis. The objective functions were formulated to minimize the difference between antiresonant frequencies observed in the host beam and the targeted frequencies, the MATLAB® `findpeaks` function is utilized in the negative FRF of the midspan displacement, in order to extract the frequencies of the antiresonant notches, while incorporating a heavy numerical penalty (P) of 1×10^6 for each missing mode. The optimization boundaries were narrowed to a specific neighborhood around X_0 using a vector of search space offsets Δ defined in Tables 4.5 and 4.6. Tables 4.7 and 4.8 summarize the optimization settings and objective functions.

Table 4.5 – Local search boundaries for the second-stage harmonic optimization in Geometry 1.

Parameter Group	Symbol	Search Range ($\pm\Delta$)
Layer set count (Integer)	N	± 2
Effective length	L	± 50 mm
Length of the weight block	LW	± 10 mm
Height of the weight block	HW	± 5 mm
Fiber angles (Integer)	$\theta 1$ and $\theta 2$	$\pm 30^\circ$
Tip width	W	± 10 mm

Table 4.6 – Local search boundaries for the second-stage harmonic optimization in Geometry 2.

Parameter Group	Symbol	Search Range ($\pm\Delta$)
Layer set count (Integer)	$NC1, NC2, N1$ and $N2$	± 2
Central beams tip width	WC	± 5
Effective lengths	$L1$ and $L2$	± 15 mm
Length of the weight block	$LW1$ and $LW2$	± 5 mm
Height of the weight blocks	$HWC, HW1, HW2$	± 5 mm
Fiber angles (Integer)	$\theta C11, \theta C12, \theta C21, \theta C22,$ $\theta 11, \theta 12, \theta 21$ and $\theta 22$	$\pm 17^\circ$
Cantilever tip width	$W1$ and $W2$	± 10 mm

Table 4.7 – GA configuration and parameters for Base Geometry 1 harmonic optimization.

Parameter	Value
Population Size	50
Max Generations	100
Crossover Fraction	0.8
Integer Variables	$N, \theta 1$ and $\theta 2$
Objective Function	$\sum_{k=1}^4 (f_{t,k} - f_{t,k})^2 + P$

Table 4.8 – GA configuration and parameters for Base Geometry 2 harmonic optimization.

Parameter	Value
Population Size	70
Max Generations	100
Crossover Fraction	0.8
Integer Variables	$NC1, \theta C11, \theta C12, NC2, \theta C21, \theta C22, N1, \theta 11, \theta 12, N2, \theta 21$ and $\theta 22$
Objective Function	$\sum_{k=1}^4 \left(\frac{f_{p,k} - f_{t,k}}{f_{t,k}} \right)^2 + P$

The values of population size and maximum number of generations were selected according to a reasonable balance between optimization time and convergence accuracy. Rao (2019) recommends population sizes between two and four times the number of design variables. In this study, at least three times the number of variables was used in all cases. The crossover fraction selected was the default value of 0.8 implemented by MATLAB®'s `ga` function.

4.3 Finite element modeling

The most sensitive part of the finite element modeling was the choice of element for the carbon fiber arm components. SOLID186, as a full second-order hexahedral element, is susceptible to shear locking in bending-dominated thin geometries, and while the mixed u-p formulation is often applied as a remedy, Bathe (2014) cautions that the equivalence between a reduced or selectively integrated displacement-based element and its mixed formulation counterpart may no longer hold when non-isotropic material laws are introduced, meaning the locking relief cannot simply be assumed and an independent verification of its validity would be required. SOLSH190 was therefore selected for the carbon fiber arm, as it is purpose-built for shell-like solid geometries and addresses transverse shear through an enhanced assumed strain formulation, avoiding reliance on the mixed formulation altogether. Although SOLID186's element reference advises against a single element through the thickness, this warning originates from shear locking concerns that SOLSH190's formulation is specifically designed to overcome; nonetheless, two elements through the thickness were still used. The remainder of the model presented no such modeling complexity, with SOLID186 applied to the isotropic solid components, where its full second-order formulation performs reliably without the complications introduced by material anisotropy. The material properties applied to the models are listed in Table 4.9

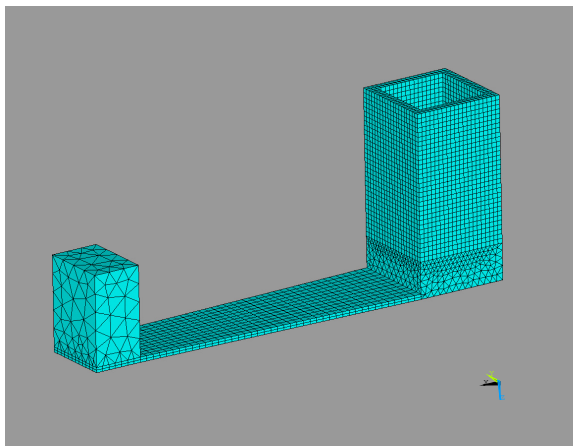
Table 4.9 – Material properties used in the finite element analysis.

Property	Aluminum 2024	Steel SAE 1020	Carbon Fiber UD (395 GPa)
E_x [GPa]	72.4	207	209
E_y [GPa]	—	—	9.45
E_z [GPa]	—	—	9.45
ν_{xy}	0.33	0.30	0.27
ν_{yz}	—	—	0.40
ν_{xz}	—	—	0.27
G_{xy} [GPa]	—	—	5.5
G_{yz} [GPa]	—	—	3.9
G_{xz} [GPa]	—	—	5.5
ρ [g/cm ³]	2.77	7.85	1.57

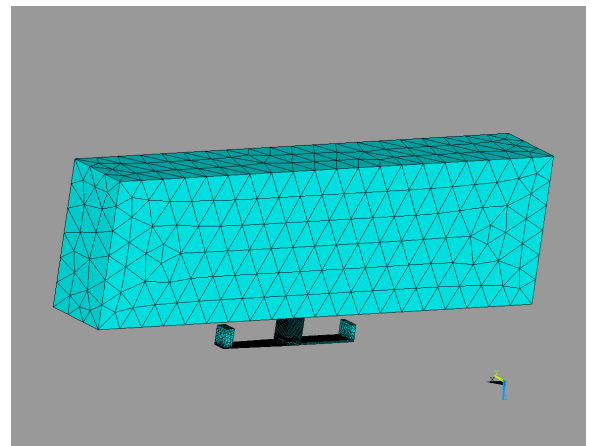
Source: [Callister Jr. \(2020\)](#) for Aluminum 2024 and Steel SAE 1020; [ANSYS, Inc. \(2023c\)](#) for Carbon Fiber UD.

4.3.1 Base Geometry 1 modeling

The finite element model of Figure 4.4 (a) was utilized in the first optimization step, where the flexural and torsional modes were tuned to each pair of target frequencies, first 11.03 Hz and 44.1 Hz, then 22.06 Hz and 111.1 Hz. Following that, the combination of the two cantilever structures was attached to the host beam as illustrated in Figure 4.4 (b) for the FRF optimization stage described in Section 4.2. The complete distribution of element types and materials is described in Table 4.10.



(a) Finite element model of a single Geometry 1 component and square hollow section connection part for modal analysis optimization step.



(b) Finite element model of a two Geometry 1 components attached to a host beam for harmonic analysis optimization step

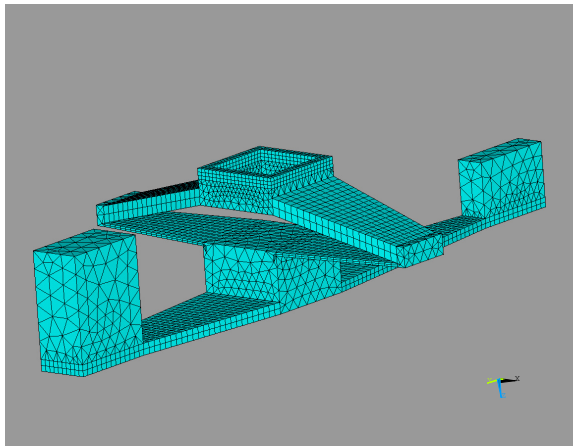
Figure 4.4 – Finite element models for Geometry 1 optimization steps.

Table 4.10 – Summary of finite element types and materials used in Base Geometry 1.

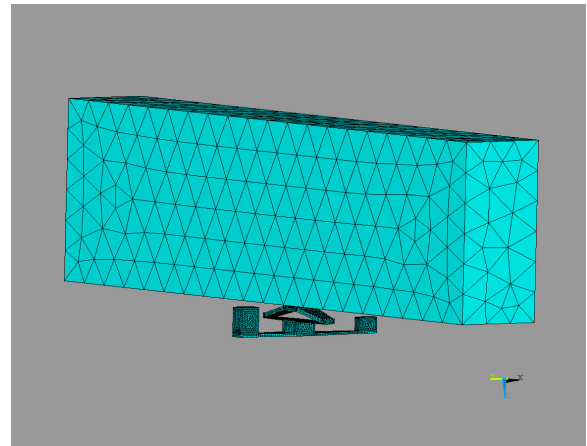
Component	Material	Element Type
Composite Beam	Carbon Fiber/Epoxy	SOLSH190
Tip Masses	Structural Steel	SOLID186 Tetrahedral
Hollow Interface	Structural Steel	SOLID186 Tetrahedral and hexahedral
Host Beam	Aluminum 2024	SOLID186 Tetrahedral

4.3.2 Base Geometry 2 modeling

In the first optimization step, utilizing the model of Figure 4.5 (a), the first four modes of vibration of the absorber were tuned to the desired target frequencies (11.03 Hz, 22.06 Hz, 44.1 Hz and 66.15 Hz) in a single optimization process. The harmonic optimization was performed in a model with the MMDVA coupled to a host beam (Figure 4.5 (b)), as was done for Geometry 1. The complete distribution of element types and materials is described in Table 4.11.



(a) Finite element model of Geometry 2 for modal analysis optimization step.



(b) Finite element of model Geometry 2 attached to a host beam for harmonic analysis optimization step.

Figure 4.5 – Finite element models for Geometry 2 optimization steps.

Table 4.11 – Summary of finite element types and materials used in Base Geometry 2.

Component	Material	Element Type
Composite Beams	Carbon Fiber/Epoxy	SOLSH190
Tip and Cenrtal Masses	Structural Steel	SOLID186 Tetrahedral
Hollow Interface	Structural Steel	SOLID186 Tetrahedral and hexahedral
Beam Connections	Structural Steel	SOLID186 Tetrahedral
Host Beam	Aluminum 2024	SOLID186 Tetrahedral

4.3.3 Mesh convergece study

To validate the finite element model, a convergence analysis was performed using the geometric parameters obtained from the modal optimization stage. The sensitivity of the first ten natural frequencies was evaluated against variations in element sizing to ensure numerical stability. The results of this analysis are presented in Figures 4.6 and 4.7. Table 4.12 lists the base element sizes used in the model, which are scaled by a sizing factor; the optimization runs reported in this study used a sizing factor of 1. The geometric parameters used for the mesh study were the optimized results after the modal optimization stage.

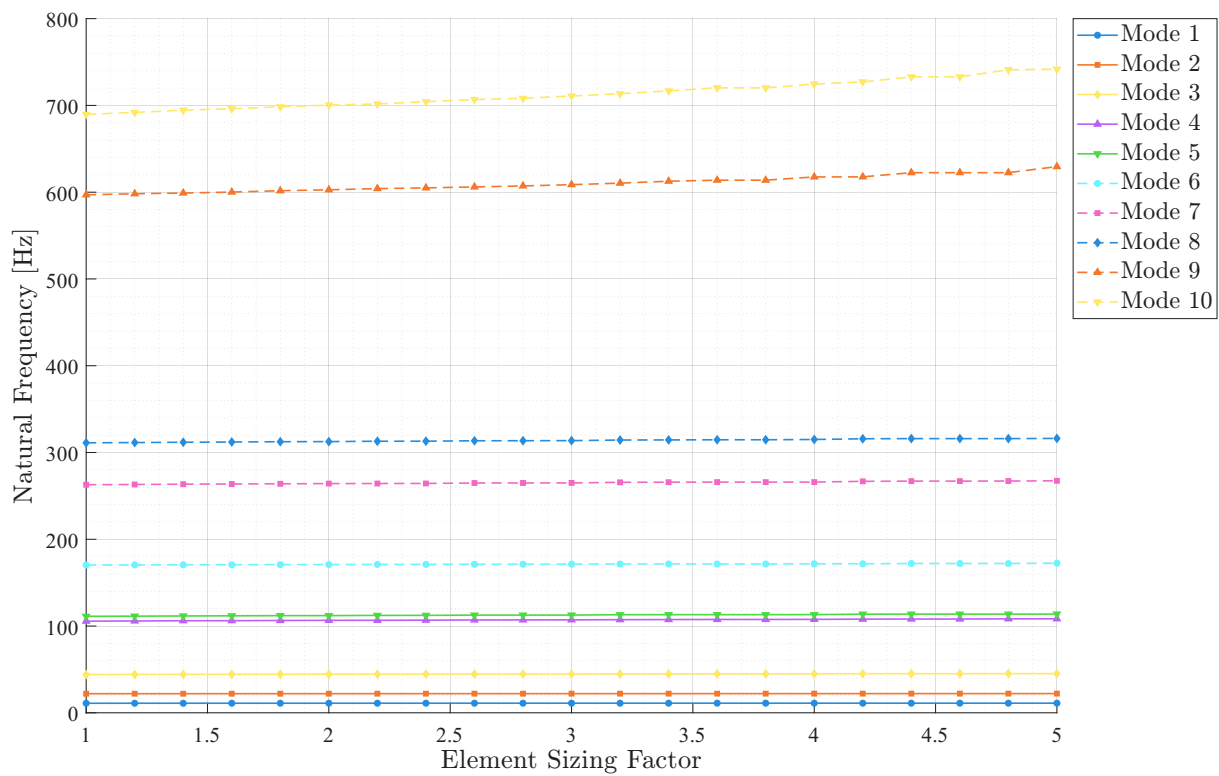


Figure 4.6 – Convergence Analysis Results for Geometry 1.

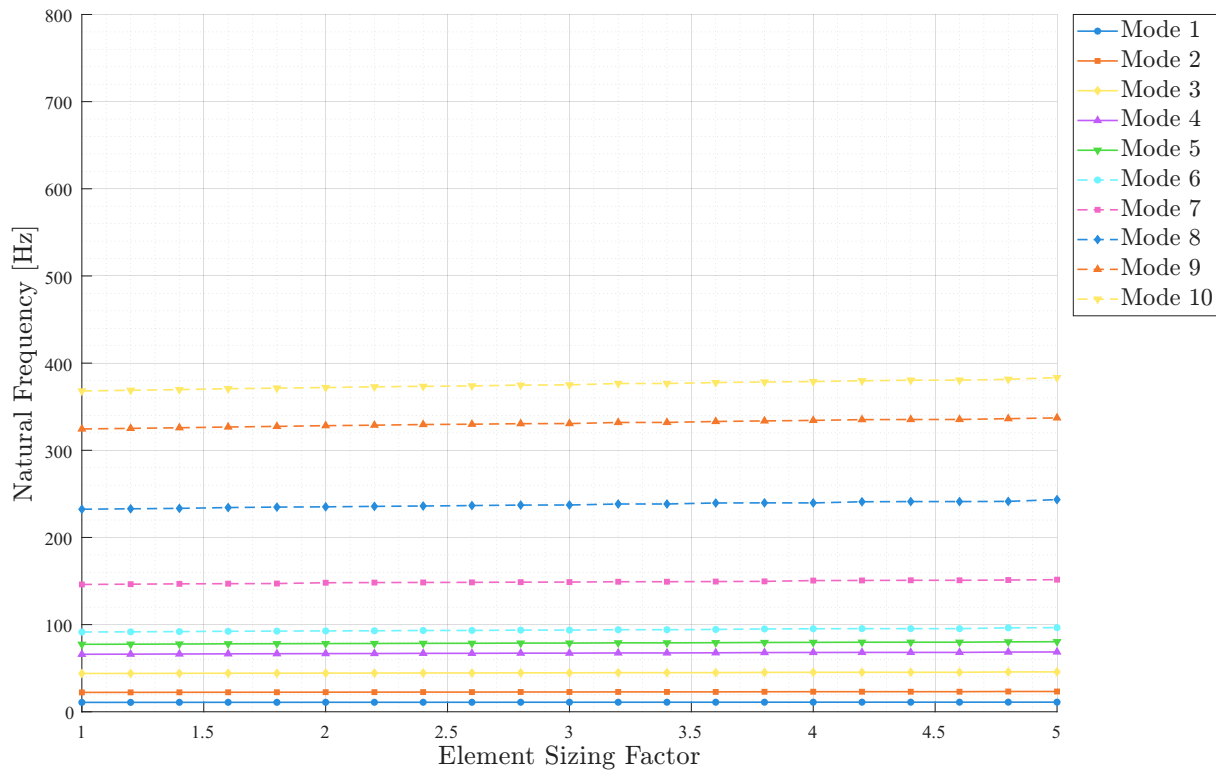


Figure 4.7 – Convergence Analysis Results for Geometry 2.

Table 4.12 – Summary of element sizes assigned via ESIZE command.

Component	Geometry 1 (mm)	Geometry 2 (mm)
Composite Beams	4	4
Central Mass Block	-	8
Cantilever Mass Blocks	10	10
Hollow Interface (Hexahedral)	2.5	2.5
Hollow Interface (Tetrahedral)	2	4
Beam Connections	-	7
Host Beam	70	70

5 RESULTS AND DISCUSSION

5.1 Base Geometry 1 results

5.1.1 Modal Optimization

The modal optimization step achieved close agreement with target frequencies as shown in Table 5.1. The resulting mode shapes are illustrated in Figure 5.1. To validate the resulting design, a harmonic analysis of the dual-branch Geometry 1 [MMDVA](#) coupled to the host beam was conducted. The midspan displacement [FRF](#) of the coupled host beam is presented in Figures 5.2 and 5.3, for three damping ratios ($\zeta = 0.5\%$, 1.0% , and 1.5%), allowing assessment of damping sensitivity. The damping ratio was implemented via `DMPRAT` command.

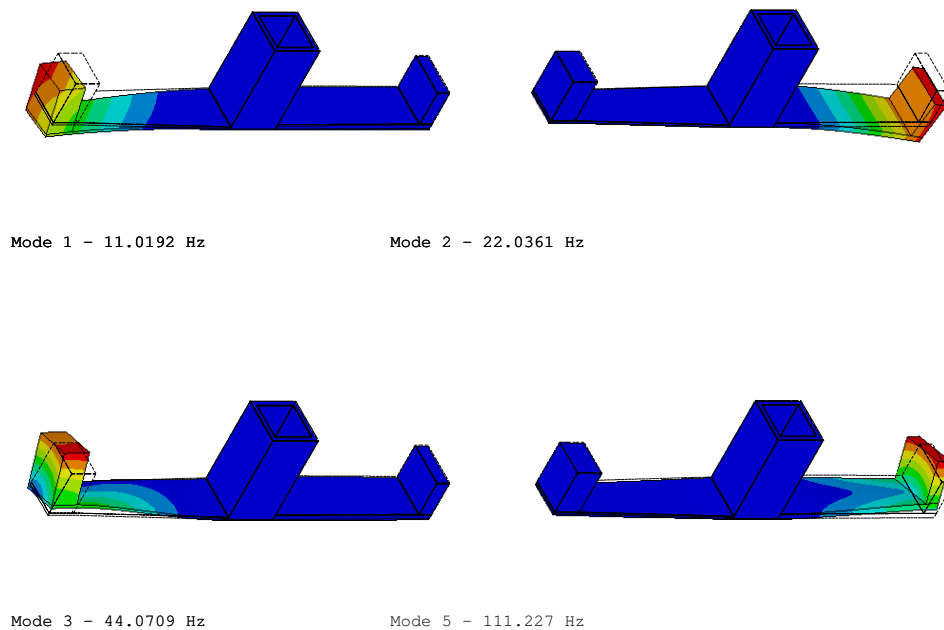
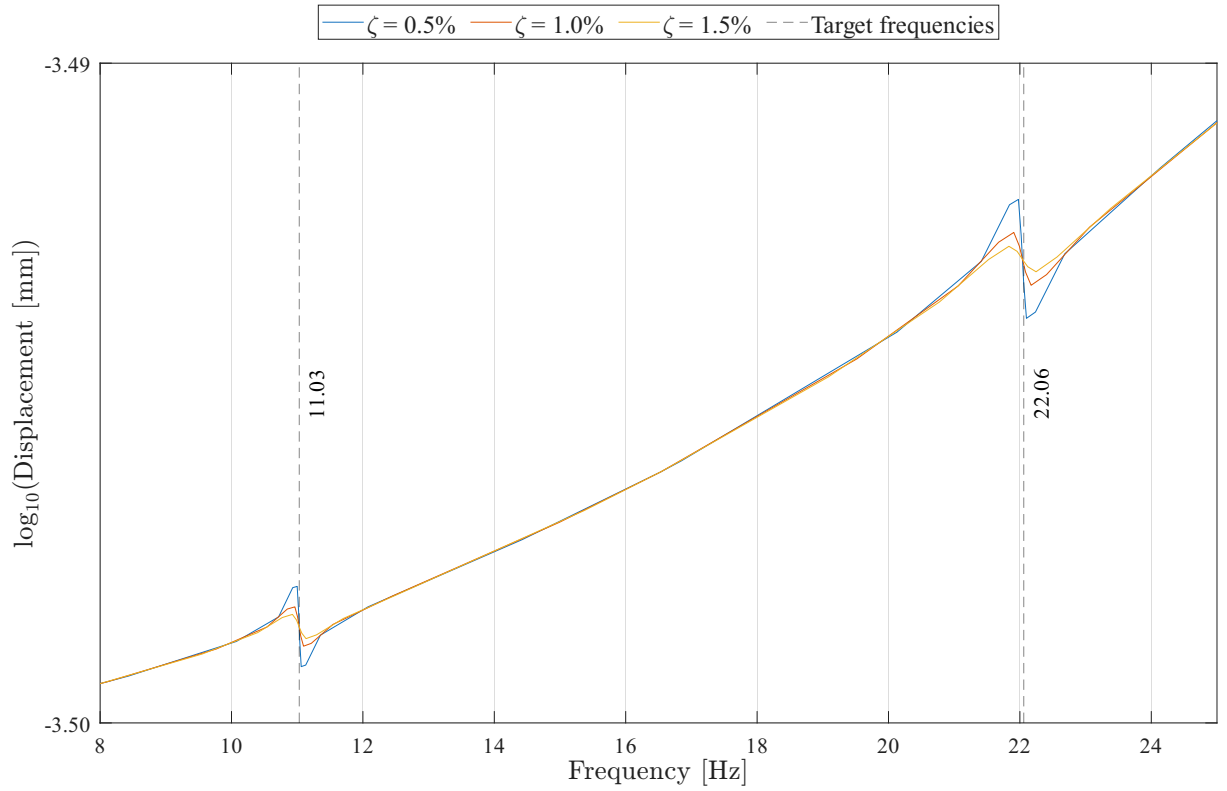


Figure 5.1 – Base Geometry 1 mode shapes.

Table 5.1 – Modal optimization results for the dual-branch Geometry 1 MMDVA.

Parameter	Unit	Side A	Side B
		11.03 Hz mode 44.1 Hz mode	22.06 Hz mode 111.07 Hz mode
N	-	7	14
θ_1	$^\circ$	19	100
θ_2	$^\circ$	27	93
L	mm	194.7339	171.9279
LW	mm	26.7642	13.8081
HW	mm	49.6971	39.8444
W	mm	39.1769	48.1559
$f_{n,1}$ (Extracted)	Hz	11.0192	22.0361
$f_{n,2}$ (Extracted)	Hz	44.0709	111.227

Figure 5.2 – Antiresonances produced in the beam by the MMDVA's flexural modes (Z -axis).

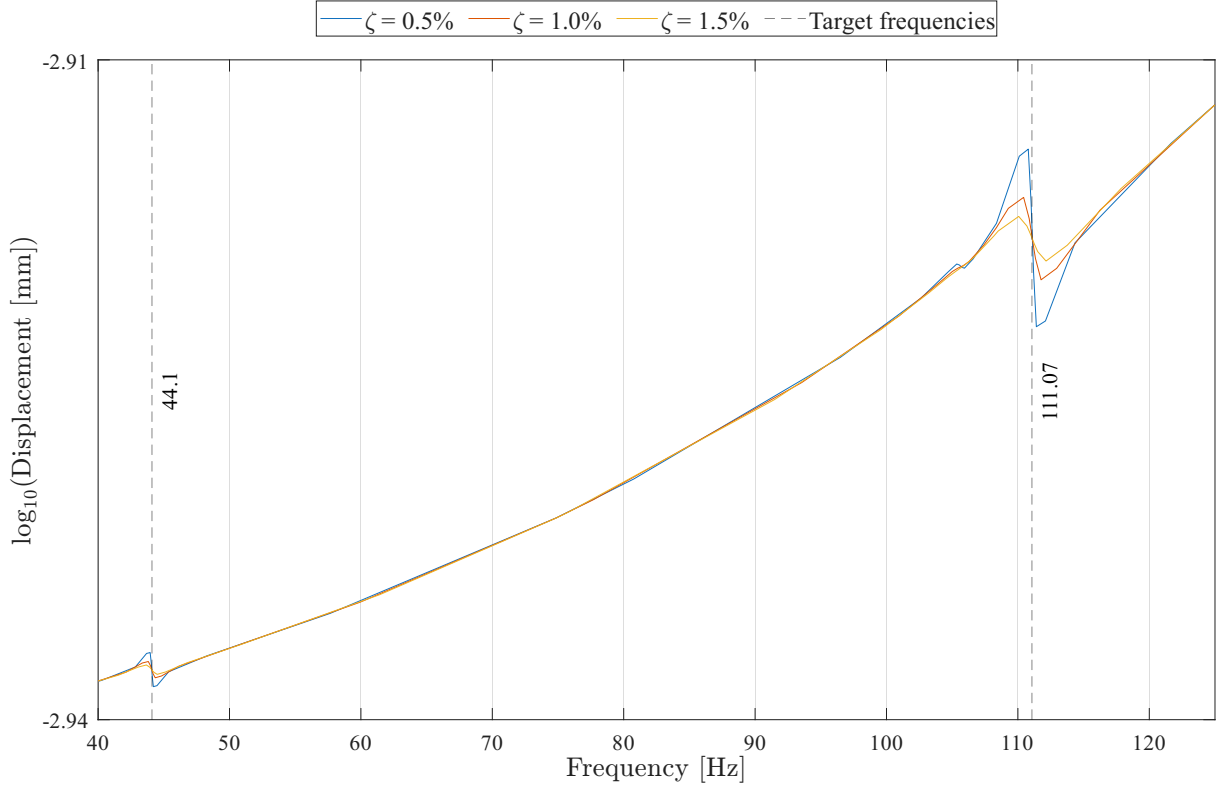


Figure 5.3 – Antiresonances produced in the beam by the MMDVA’s torsional modes (Y-axis).

As shown in the panels, the antiresonant notches shift towards higher frequencies as structural damping increases. While this frequency shifting effect is expected, increasing damping significantly reduces the depth of the notches, indicating a trade-off between frequency bandwidth and amplitude reduction.

5.1.2 Harmonic Optimization

Following the identification of optimal parameters from the initial modal analysis, a second-stage refinement was implemented using harmonic analysis coupled with the host structure as described in Section 4.2. The results, summarized in Table 5.2 and illustrated in Figures 5.4 and 5.5, demonstrate the effectiveness of the sequential optimization strategy in accounting for structural damping and system coupling. The modes shapes from the modal step were preserved, as shown in Figure 5.6.

A structural damping ratio of 0.5 % was selected to run the harmonic optimization. This conservative lower-bound assumption aligns with classical DVA design philosophy, where effectiveness relies on minimal inherent damping to ensure deep antiresonance notches. As Bramwell notes regarding fuselage-mounted mass-spring absorbers: "to be effective the inherent damping of the moving mass must be minimal so that the amplification at resonance is large" (BRAMWELL; BALMFORD; DONE, 2001). Damping ratios in the range of 0.5 % to 0.8 % are reported by Feng *et al.* (2025) for carbon fiber laminates.

Nevertheless, experimental data for the specific geometries and materials considered here should be evaluated in future studies.

Table 5.2 – Harmonic optimization results for the dual-branch Geometry 1 MMDVA.

Parameter	Unit	Side A	Side B
		11.03 Hz mode 44.1 Hz mode	22.06 Hz mode 111.07 Hz mode
N	-	7	14
θ_1	$^\circ$	19	100
θ_2	$^\circ$	27	93
L	mm	194.9213	171.7862
LW	mm	26.7839	13.8365
HW	mm	49.7267	39.8524
W	mm	39.2718	48.5420

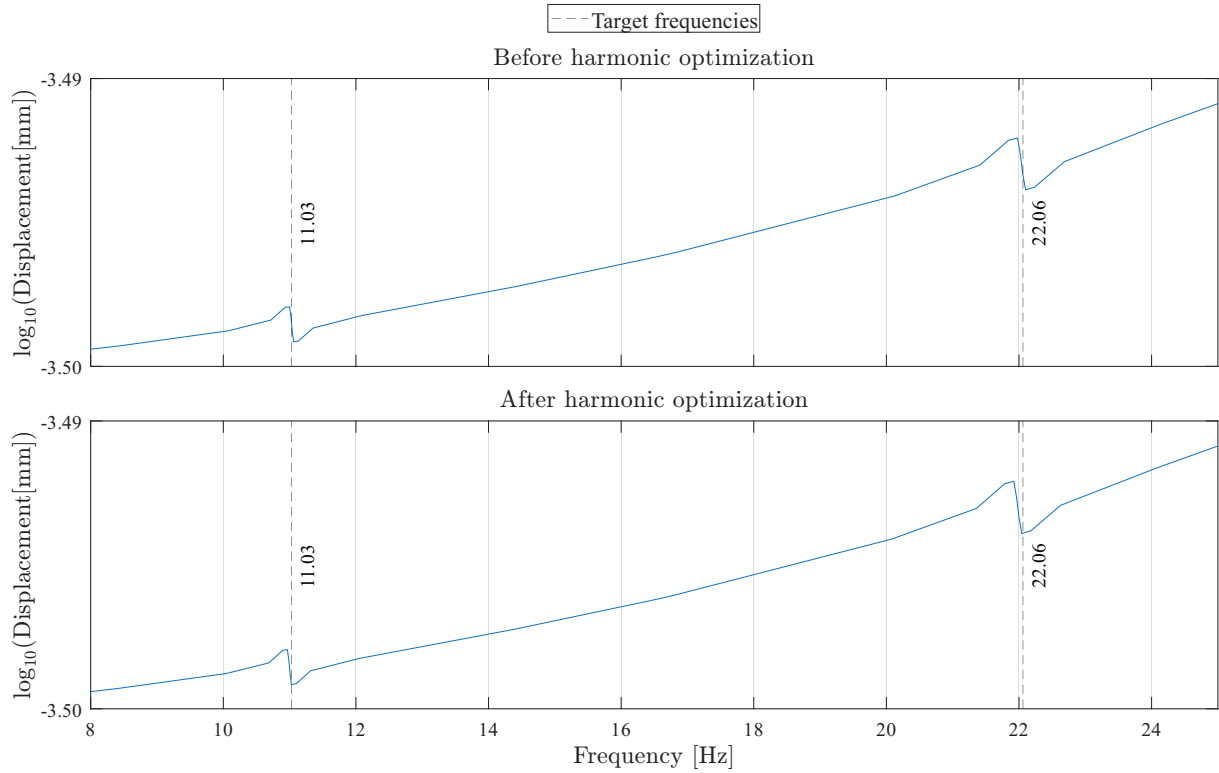


Figure 5.4 – Comparison of antiresonances caused by flexural modes (Z -axis).

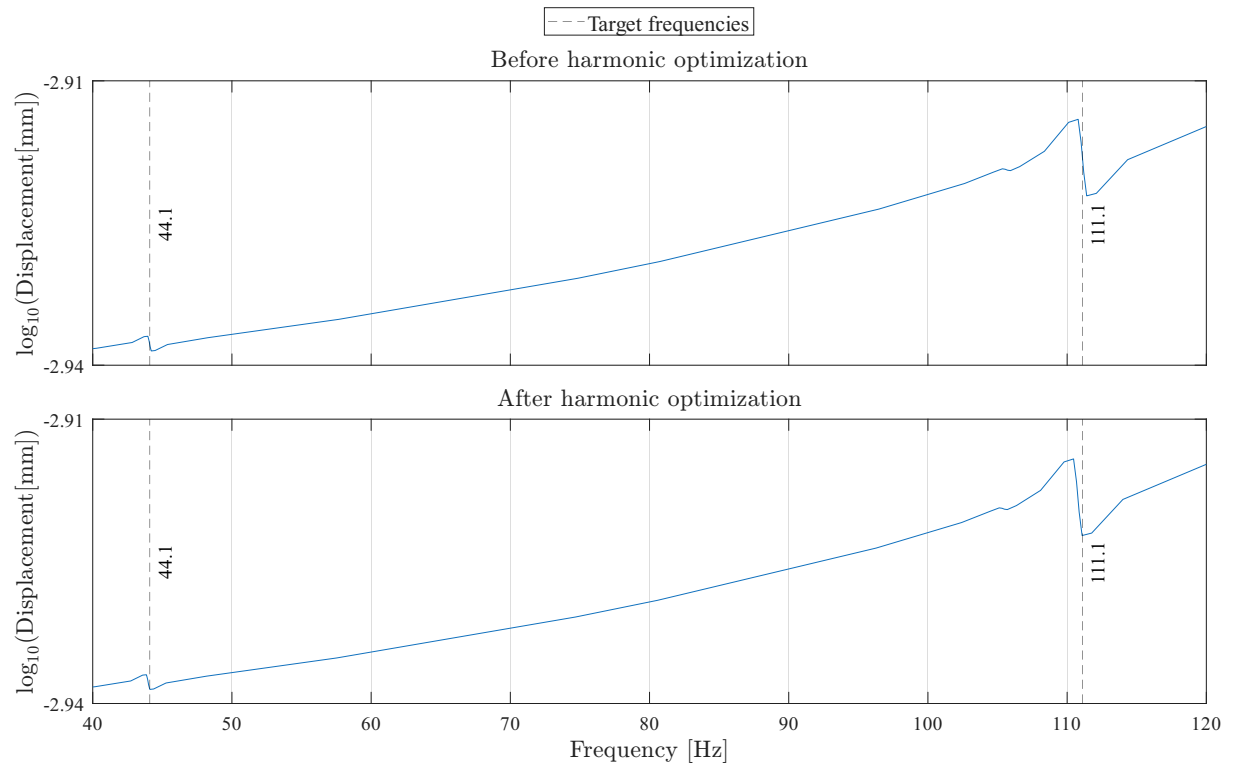


Figure 5.5 – Comparison of antiresonances caused by torsional modes (Y-axis).

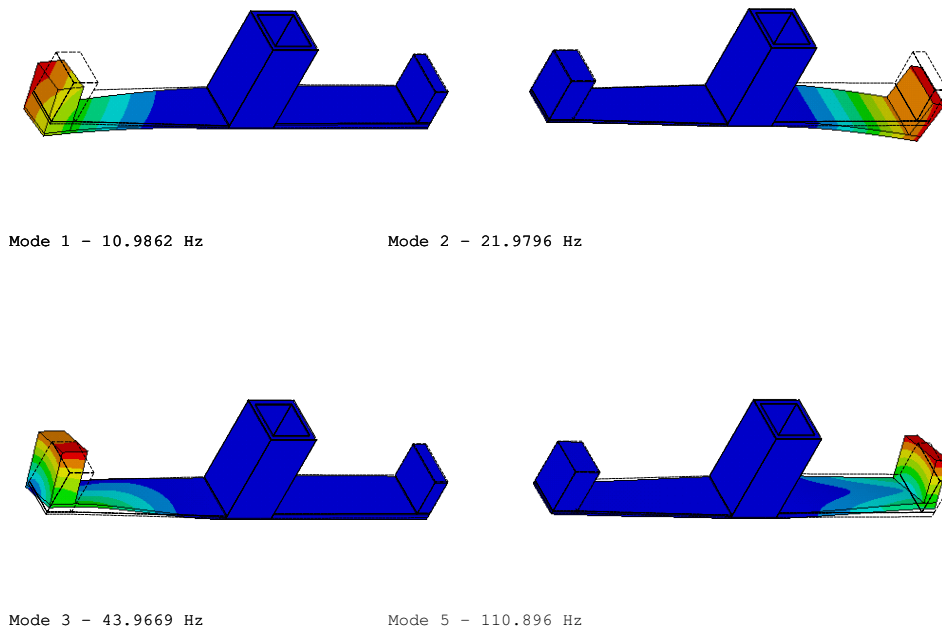


Figure 5.6 – Base Geometry 1 mode shapes after harmonic optimization.

5.2 Base Geometry 2 results

5.2.1 Modal Optimization

The mode shapes illustrated in Figure 5.7 reveal that each weight block contributes meaningfully to the targeted deformation: Mode 2 and Mode 3 exhibit predominantly flexural motion along the Z-axis, while Mode 1 shows torsional rocking behavior of the cantilever masses. Mode 4 captures a coupling between bending and torsion. The optimized parameters are listed in Table 5.3, although frequency convergence was less precise than in Geometry 1, the result provides a suitable starting point for harmonic refinement. The damping factor selected for the harmonic analysis was 0.5% based on the analysis that was made for Geometry 1 in Section 5.1.2. The FRF results of the coupled system (MMDVA and beam) show that the antiresonant notches appear along Z-axis as displayed in Figure 5.8.

Table 5.3 – Modal optimization results for the Geometry 2 MMDVA.

Central Component			Beam 1			Beam 2		
Param.	Unit	Value	Param.	Unit	Value	Param.	Unit	Value
$NC1$	-	23	$N1$	-	16	$N2$	-	13
$\theta C11$	°	27	$\theta 11$	°	116	$\theta 21$	°	29
$\theta C12$	°	149	$\theta 12$	°	122	$\theta 22$	°	76
$NC2$	-	5	$L1$	mm	147.1776	$L2$	mm	185.6106
$\theta C21$	°	61	$LW1$	mm	49.2641	$LW2$	mm	48.6617
$\theta C22$	°	132	$HW1$	mm	58.6472	$HW2$	mm	30.0709
WC	mm	31.2077	$W1$	mm	28.3952	$W2$	mm	15.6690
HWC	mm	33.3691						
Extracted Natural Frequencies								
$f_{n,1} = 10.773 \text{ Hz}$			$f_{n,2} = 22.2405 \text{ Hz}$					
$f_{n,3} = 43.928 \text{ Hz}$			$f_{n,4} = 65.9678 \text{ Hz}$					

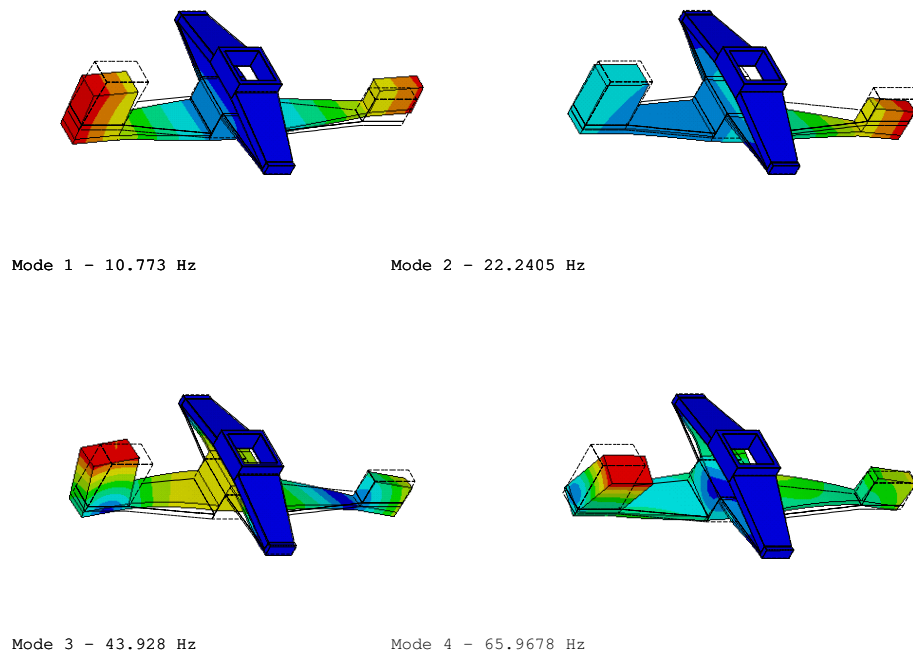


Figure 5.7 – Base Geometry 2 mode shapes.

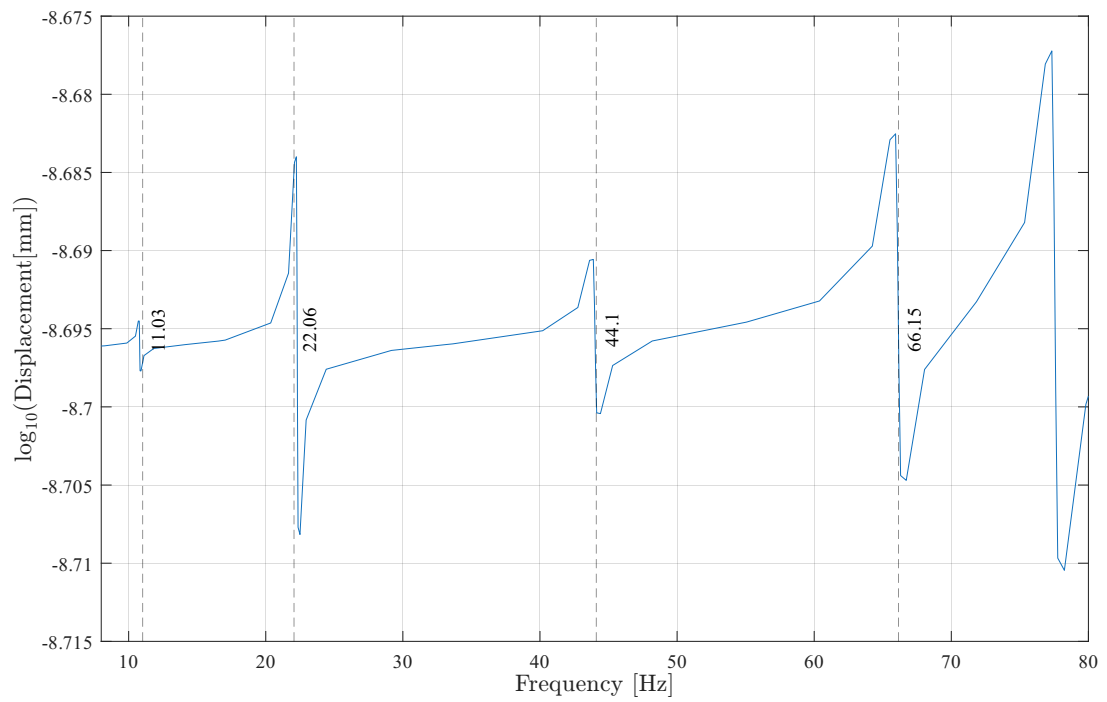


Figure 5.8 – Antiresonances produced in the beam by the MMDVA (Z-axis).

5.2.2 Harmonic Optimization

Just as in Geometry 1, the harmonic optimization step improved the alignment of antiresonances with desired frequencies while maintaining a similar FRF. The final parameter values are listed in Table 5.4. Figure 5.9 shows the improvement of the FRF compared to the modal optimization results. The mode shapes remained the same as illustrated in Figure 5.10.

Table 5.4 – Harmonic optimization results for the Geometry 2 MMDVA.

Central Component			Beam 1			Beam 2		
Param.	Unit	Value	Param.	Unit	Value	Param.	Unit	Value
$NC1$	-	22	$N1$	-	16	$N2$	-	13
$\theta C11$	$^\circ$	27	$\theta 11$	$^\circ$	121	$\theta 21$	$^\circ$	31
$\theta C12$	$^\circ$	148	$\theta 12$	$^\circ$	115	$\theta 22$	$^\circ$	75
$NC2$	-	5	$L1$	mm	145.5340	$L2$	mm	186.5810
$\theta C21$	$^\circ$	60	$LW1$	mm	48.2563	$LW2$	mm	48.6134
$\theta C22$	$^\circ$	132	$HW1$	mm	58.6716	$HW2$	mm	30.0788
WC	mm	31.3504	$W1$	mm	28.1998	$W2$	mm	16.0069
HWC	mm	34.0139						

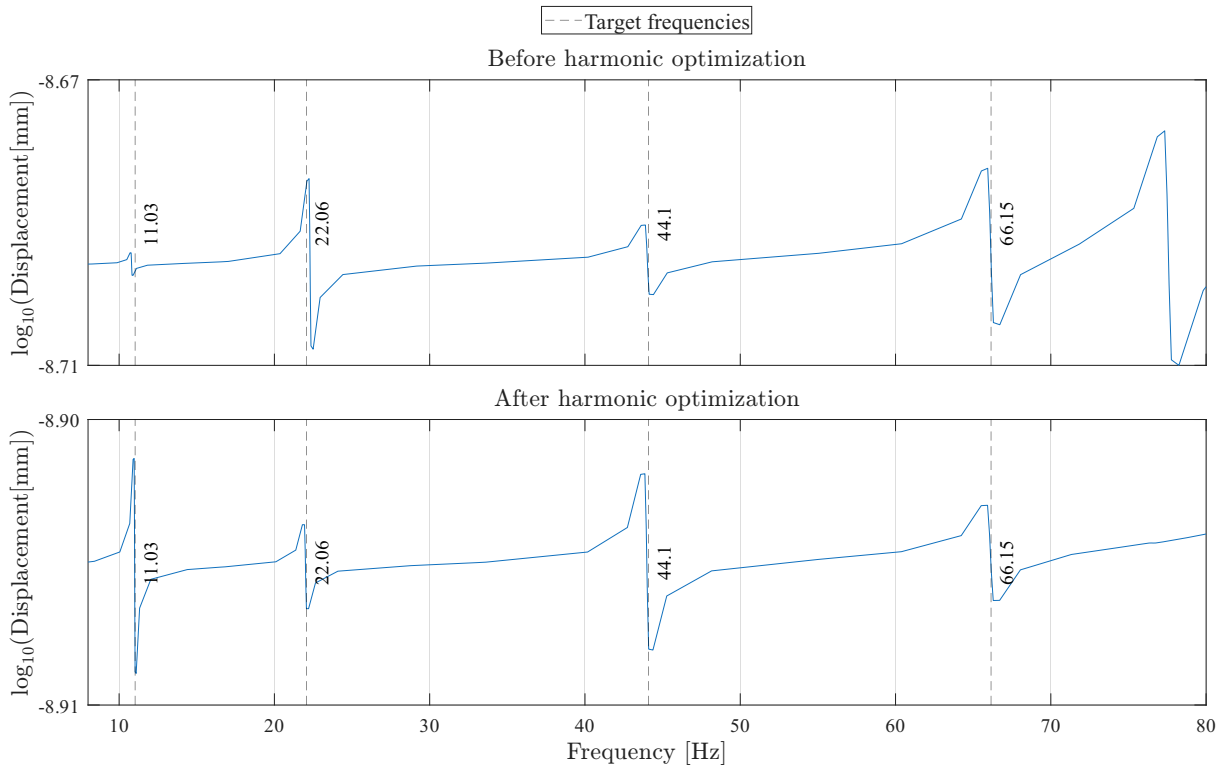


Figure 5.9 – Comparison of antiresonances produced in the beam by the MMDVAs (Z-axis).

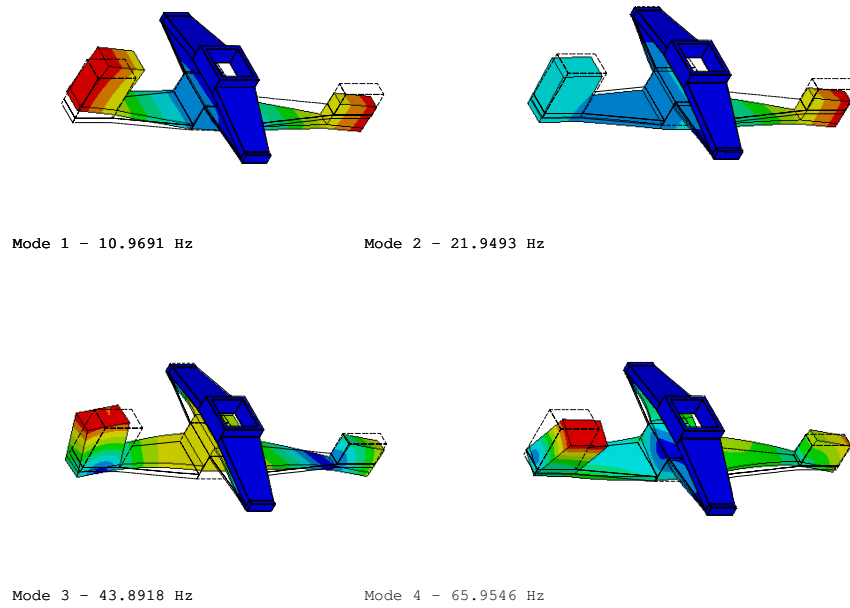


Figure 5.10 – Base Geometry 2 mode shapes after harmonic optimization.

5.3 Fitness convergence analysis

The convergence history of each [GA](#) run was tracked by recording the best penalty (fitness) score at every generation. These convergence curves provide insight into how efficiently the optimizer located low-penalty designs and whether the algorithm was allowed to run long enough to reach a stable optimum. Figures [5.11](#) and [5.13](#) report the penalty values across generations for the modal optimization of Geometries 1 and 2, respectively. Both cases still show occasional improvements in the final generations rather than a fully flat trend, suggesting that additional generations could yield further gains; nevertheless, the results obtained were accurate enough to proceed to the next optimization step. Figure [5.12](#) shows that the harmonic optimization for Geometry 1 reached a stable, near-flat penalty in the final generations, whereas Figure [5.14](#) was terminated by the maximum-generation limit before its own stall-based stopping criterion was met, indicating that Geometry 2 could likely benefit from additional generations to fully converge.

Regarding simulation time, after configuring optimal simulation and parallel computing settings, optimization times of around 8 hours were achievable using an Intel® Core i9-14900 processor and 64 GB of RAM; however, this can extend to days or weeks with a poorly configured or outdated system. In the [APDL](#) script, it is essential to disable the harmonic solution expansion pass.

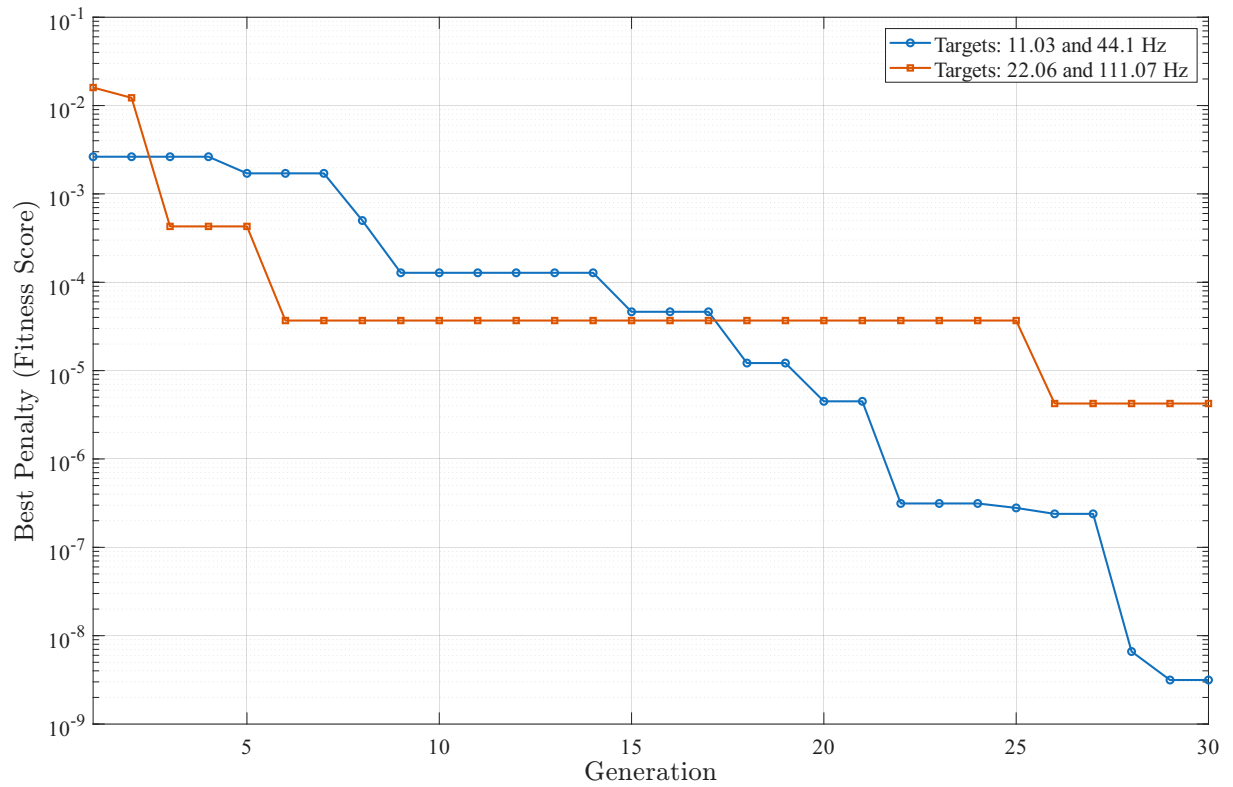


Figure 5.11 – Optimization scores for Geometry 1 modal optimization

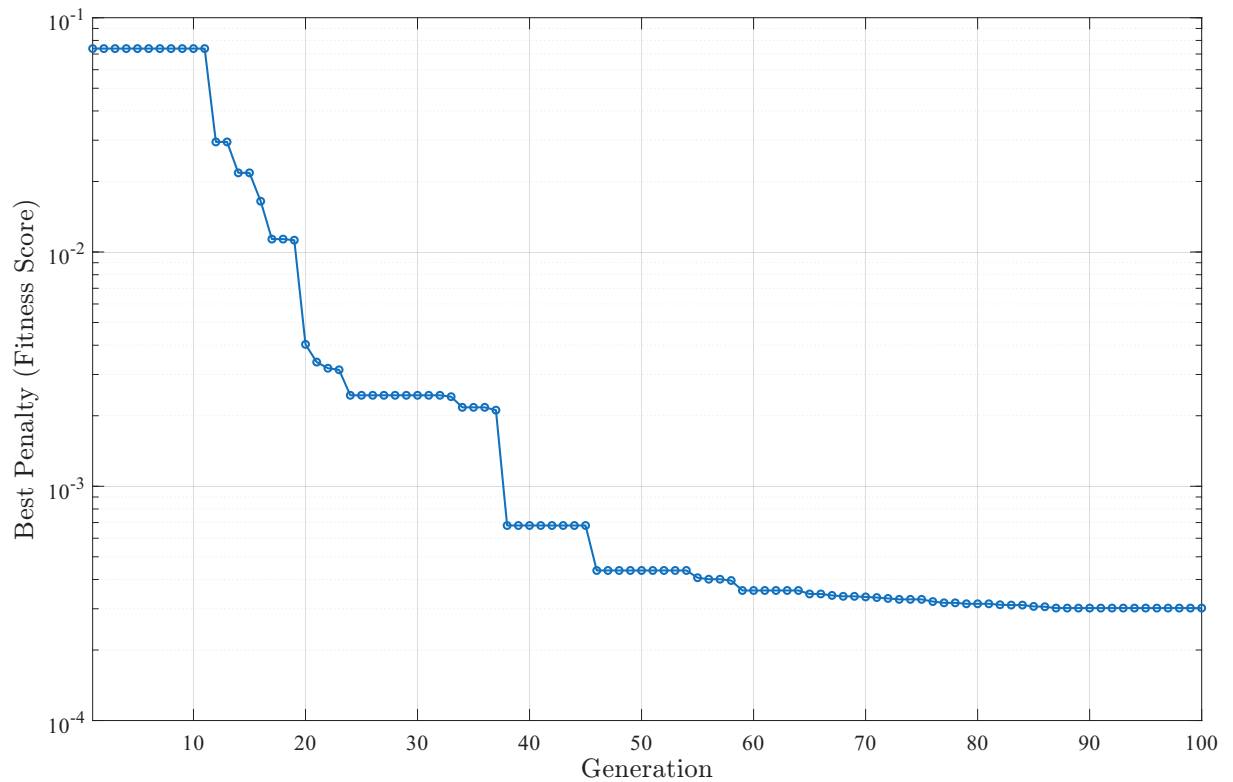


Figure 5.12 – Optimization scores for Geometry 1 harmonic optimization

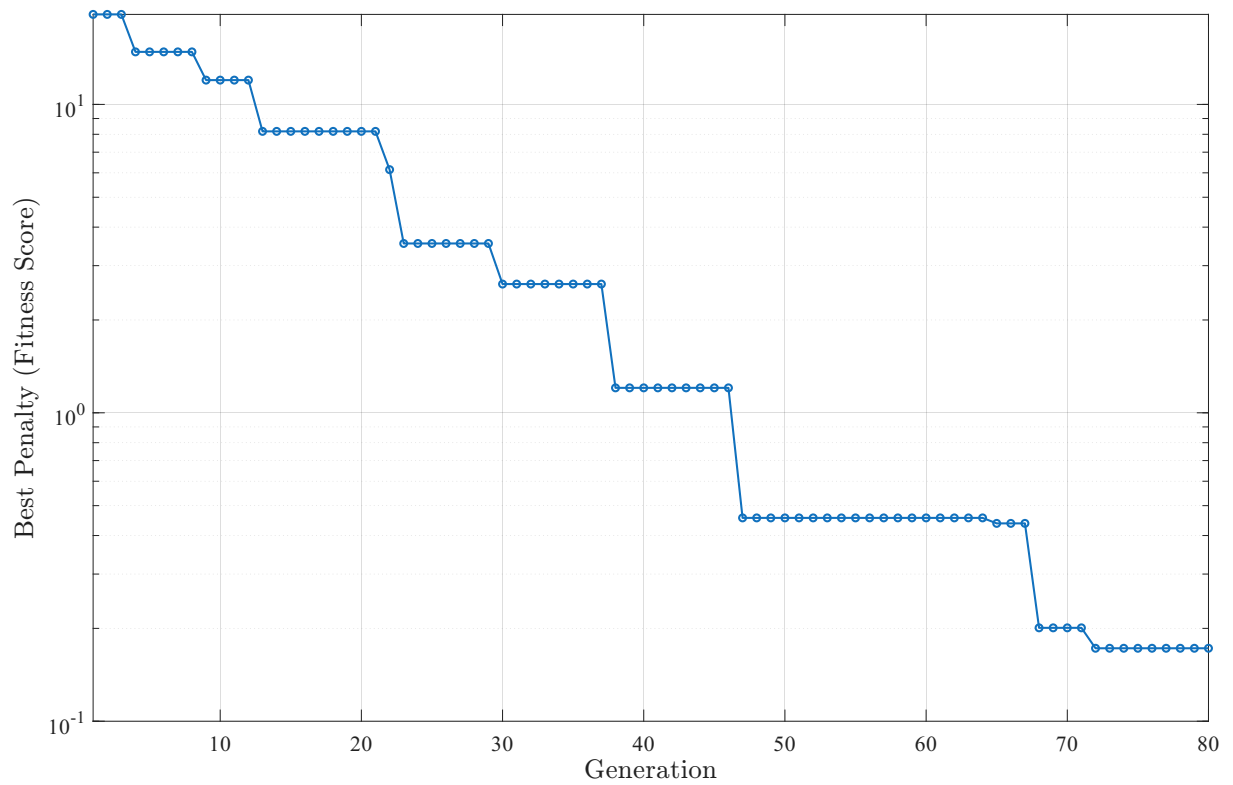


Figure 5.13 – Optimization scores for Geometry 2 modal optimization

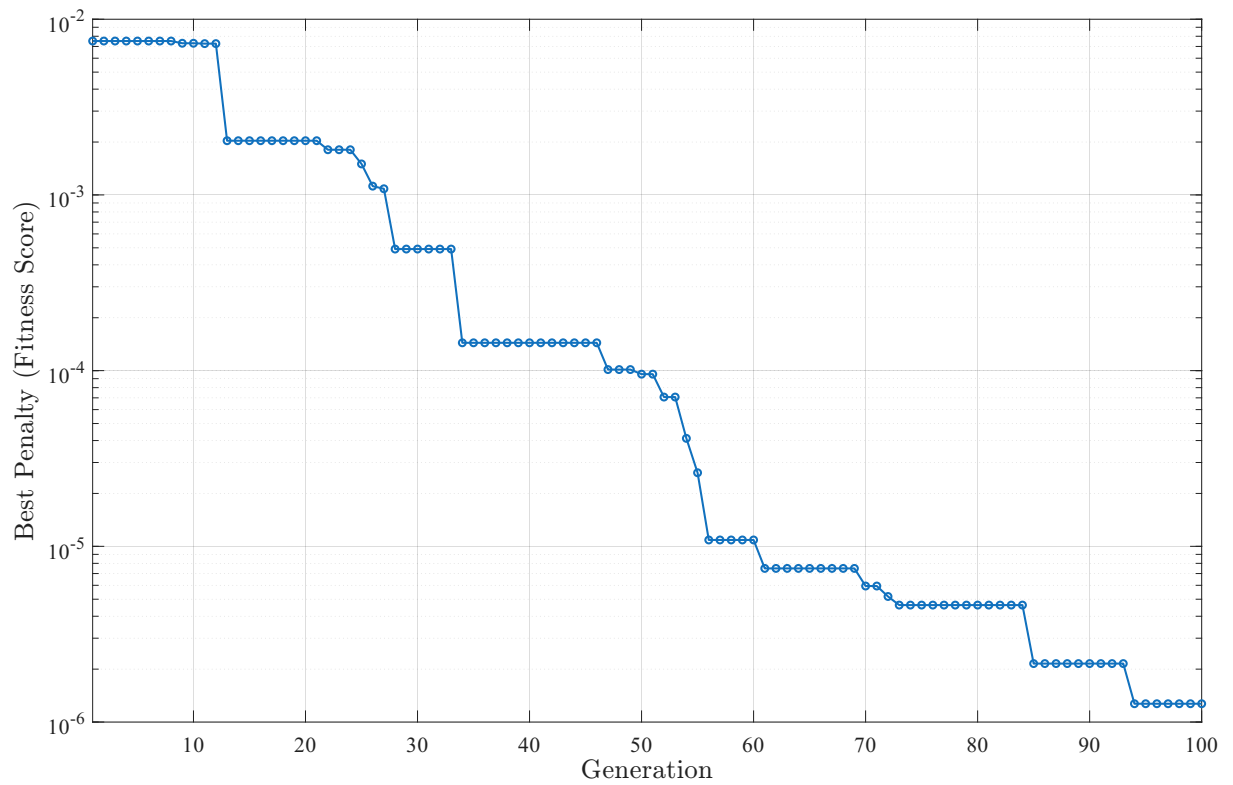


Figure 5.14 – Optimization scores for Geometry 2 harmonic optimization

5.4 Comparative analysis

Both geometries achieved comparable frequency convergence through the two-stage optimization procedure, demonstrating that the sequential modal-harmonic strategy is robust across different structural configurations. However, the geometries differ meaningfully in performance characteristics. Geometry 2 concentrated the antiresonances along a single axis, a relevant advantage given that the vertical axis consistently exhibited higher acceleration levels in the accelerometer measurements, as shown in Figure 3.3. This improvement, however, comes at the cost of a narrower effective bandwidth and greater complexity and physical footprint. Geometry 1, in contrast, offers a more practical design: its compact form factor, broader bandwidth, and simpler configuration make it better suited for space-constrained applications where robustness to frequency uncertainty is prioritized over peak attenuation. It is also worth noting that achieving the 11 Hz natural frequency already required a 2.8 mm thick laminate for Geometry 1 and 2 mm for Geometry 2, raising concerns about structural fragility and fatigue life; extending the design toward 5.5 Hz was therefore deliberately avoided. The choice between the two configurations ultimately depends on application requirements: Geometry 2 is preferable when precise narrow-band attenuation along the vertical axis is critical, while Geometry 1 is more appropriate when compactness and frequency robustness are paramount.

Another important analysis is the weight distribution. For Geometry 1, the final parameters resulted in a steel block mass of 0.621 kg and a carbon fiber spring mass of 0.112 kg. For Geometry 2, the corresponding values were 1.426 kg and 0.199 kg. These values are good for initial prototyping and experimental studies, but should be scaled to a higher mass ratio in relation to the helicopter; the current Lynx's flexispring absorber has an effective mass around 45 kg according to [White \(1979\)](#). In order to take full advantage of the carbon fiber, the boundaries and objectives should be adjusted to achieve a smaller ratio between spring mass and total mass since the flexispring already presents 4 %, while the results were 15 % for Geometry 1 and 12 % for Geometry 2.

6 CONCLUSIONS

This work began with a review of the helicopter vibration literature, covering the main excitation sources, their frequency content, and the passive and active attenuation strategies currently employed in rotorcraft applications. The review also encompassed [DVA](#) and [MMDVA](#) literature, highlighting the distinctions between helicopter applications and other engineering contexts. Particularly the requirement to attenuate rotor harmonics at fixed excitation frequencies, rather than shifting or avoiding structural natural frequencies, and the strict weight penalty concerns imposed by airframe integration. Experimental accelerometer data acquired from a helicopter airframe provided the frequency content that informed the target frequencies for the [MMDVA](#) design, confirming that harmonics other than the blade passage frequency contribute significantly to the vibration environment and must be addressed by any effective attenuation strategy.

Two original geometries were proposed and successfully modeled using [FEM](#) with variable geometric parameters, enabling a flexible and parametric design framework. A MATLAB® based optimization routine was implemented, leveraging parallel computing to reduce the computational cost of the tuning process. A two-stage optimization strategy, combining modal and harmonic analyses, was developed and applied to both geometries. The resulting [FRF](#) of the host structure coupled to the [MMDVA](#) exhibited the characteristic antiresonance pattern predicted by lumped-parameter [DVA](#) theory discussed in Chapter 2, confirming that the continuous elastic device replicates the essential dynamic behavior of its theoretical counterpart, but can also take advantage of multiple mode shapes to create antiresonances. In light of the results obtained, this research supports the following specific conclusions:

- The experimental characterization of the helicopter airframe revealed that, while the [BPF](#) represents the dominant excitation, other harmonics contribute significantly to the overall vibration environment, reinforcing the motivation for a multi-frequency attenuation strategy;
- The exploitation of multiple mode shapes (flexural and torsional) was successfully demonstrated for both geometries, enabling simultaneous vibration attenuation along multiple axes;
- The two-stage optimization strategy proved effective, with the harmonic refinement step being necessary to account for structural coupling and damping effects that the modal optimization alone could not capture;
- As shown in Section 5.1, Figures 5.2 and 5.3, structural damping exerts a stronger influence on antiresonance bandwidth than geometric parameters alone. This sug-

gests that damping could be treated as an active design parameter. For instance, through controlled variation of viscoelastic layer thickness;

- The harmonic optimization was conducted using a structural damping ratio of 0.5 %. While this conservative assumption facilitated the extraction of deep antiresonance notches, it may not accurately represent the real-world structural behavior of a composite-steel helicopter airframe, and should be revisited once experimental damping measurements are available;
- Both geometries achieved comparable frequency convergence. Geometry 2 produced deeper antiresonance notches at the cost of narrower bandwidth and greater geometric complexity, while Geometry 1 offers a more compact solution with broader bandwidth.

The following directions are suggested for future work:

- Experimental validation using real manufactured parts is the most immediate need, allowing assessment of manufacturing tolerances, damping ratio, material uncertainties, and boundary condition effects on [MMDVA](#) performance. In this context, additive manufacturing presents a promising pathway for producing the carbon fiber springs, either through direct printing of continuous or chopped carbon fiber reinforced. This approach would enable rapid design iteration and geometric customization at low cost, substantially reducing the time and expense associated with iterative prototyping and experimental validation cycles.
- Reformulation of the problem as a multiobjective optimization, seeking alignment of the antiresonant frequencies with the target values while keeping the mass ratio close to a value compatible with payload constraints. As shown by Equation 2.5, the mass ratio directly affects the antiresonance depth, which justifies its inclusion as a design objective. However, this reformulation is expected to increase the computational cost, requiring a larger number of iterations and function evaluations for the optimization algorithm to approximate the Pareto front.
- Alternative geometries should be explored to further expand the design space and investigate potential trade-offs between bandwidth, attenuation depth, and compactness;
- The incorporation of viscoelastic layers into the [MMDVA](#) structure represents a promising direction for passive damping control without a significant increase in structural complexity. Furthermore, it enables a specific bandwidth to be targeted as an optimization objective, using the viscoelastic layer thickness as a control variable.

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- The passive attenuation approach could be extended into a hybrid active-passive framework, where force actuators apply a constant preload to shift the natural frequencies of the device, enabling real-time tuning to accommodate varying operating conditions.

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